

# String GUTs

G. Aldazabal <sup>\*1</sup>, A. Font<sup>2</sup>, L.E. Ibáñez<sup>1</sup> and A. M. Uranga<sup>1</sup>

<sup>1</sup>Departamento de Física Teórica,  
Universidad Autónoma de Madrid,  
Cantoblanco, 28049 Madrid, Spain.

<sup>2</sup>Departamento de Física, Facultad de Ciencias,  
Universidad Central de Venezuela,  
A.P. 20513, Caracas 1020-A, Venezuela.

## Abstract

Standard SUSY-GUTs such as those based on  $SU(5)$  or  $SO(10)$  lead to predictions for the values of  $\alpha_s$  and  $\sin^2\theta_W$  in amazing agreement with experiment. In this article we investigate how these models may be obtained from string theory, thus bringing them into the only known consistent framework for quantum gravity. String models with matter in standard GUT representations require the realization of affine Lie algebras at higher levels. We start by describing some methods to build level  $k = 2$  orbifold string models with gauge groups  $SU(5)$  or  $SO(10)$ . We present several examples and identify generic features of the type of models constructed. Chiral fields appropriate to break the symmetry down to the standard model generically appear in the massless spectrum. However, unlike in standard SUSY-GUTs, they often behave as string moduli, i.e., they do not have self-couplings. We also discuss briefly the doublet-triplet Higgs splitting. We find that, in some models, built-in sliding-singlet type of couplings exist.

---

<sup>\*</sup>Permanent Institutions: CNEA, Centro Atómico Bariloche, 8400 S.C. de Bariloche, and CON-ICET, Argentina

# 1 Introduction

There has been recently a renewed interest in the field of supersymmetric grand unified theories (SUSY-GUTs) [1]. One of the justifications for that interest is the very good agreement found between the predicted [2] values for  $\sin^2\theta_W$  and  $\alpha_s$  and the values experimentally measured, particularly at LEP [3].

We know that SUSY-GUTs by themselves cannot be the whole story since they do not address the problem of quantum gravity. On the other hand, supersymmetric 4-D strings [4] provide a general framework for the unification of all interactions including gravity into a finite theory. Thus an obvious possibility is trying to combine both elements and construct 4-D strings whose massless sector resemble SUSY-GUTs. We will call this type of structure *String GUTs*.

Although this is an obvious idea, in the literature there are only a few attempts [5, 6, 7, 8, 9] at developing it. The reason for this situation is twofold:

- i. A fundamental ingredient in any SUSY-GUT is the set of chiral fields that break the GUT symmetry down to the standard model, the GUT-Higgs fields. In  $SU(5)$  the simplest option is an adjoint 24-plet. In  $SO(10)$  one needs two set of Higgses, apart from the adjoint 45 (or the symmetric 54 representation) one needs other Higgs fields, such as  $16 + \bar{16}$  or  $126 + \bar{126}$ , to lower the rank. However, in the most common type of 4-D strings built in the past, no adjoints (nor 54s in the  $SO(10)$  case) may be present in the spectrum of the theory since the affine Lie gauge algebra is realized at level  $k = 1$ . Thus, models with the gauge group realized at  $k \geq 2$  must be considered. This turns out to be technically non-trivial due to the constraints of modular invariance.
- ii. It is not obvious that string-GUTs give us any phenomenological improvement over the  $k = 1$  4-D string models already constructed in the past. Indeed, the fact that the gauge couplings unify at a single scale is also present in strings [10] even without any GUT symmetry present, i.e. in a  $k = 1$  4-D string with the SM gauge group. Another prominent virtue of GUTs, charge quantization, may also be seen as a consequence of the cancellation of anomalies so that it is not necessary to invoke any GUT symmetry to explain it. In addition SUSY-GUTs are not free of some important problems, in particular the doublet-triplet splitting problem which has already been with us for more than a decade [1]. This problem is on the other hand not necessarily present in a 4-D string with SM gauge group.

It is perhaps time to reconsider the above two points since the fantastic agreement between theory and experiment for the joining of coupling constants may turn out not to be just a coincidence. Although couplings are also unified in e.g. a string with a SM gauge group, the unification scale is  $M_{string}$  [11] which is around a factor 20 larger than the value  $M_X \simeq 10^{16}$  GeV suggested by the extrapolation of the low energy coupling constants. This in turn leads to numerical results for  $\sin^2\theta_W$  and  $\alpha_s$  in disagreement with data [12, 13]. This factor 20 may be explained by different effects like large string threshold corrections [12, 14], existence of extra matter fields beyond those of the MSSM [13] or a non-canonical value for the normalization  $k_1$  corresponding to the  $U(1)$  hypercharge [15]. Although indeed all or some of these effects may be present one must admit that then one is really adjusting, not predicting, the value of  $\sin^2\theta_W$  and  $\alpha_s$ .

We think that, in view of the above arguments, it is worth attempting to construct GUTs from strings. Even from the merely technical string point of view it is interesting to construct higher level 4-D string models in order to study their generic

properties as compared to the  $k = 1$  models constructed in the past. We address in this article the construction of higher level string GUTs by using 4-D orbifold techniques. To the best of our knowledge only Ref. [6] has previously dealt with the problem of constructing orbifold models with higher level affine Lie algebras. There it is explained how three different methods can be used to construct 4-D orbifolds with some group factor realized at higher level. In particular, an specific  $Z_3$  orbifold toy-model with an  $SU(3)$  group realized at level  $k = 3$  was constructed simultaneously by the same three methods that will be used in the following.

The structure of this paper is as follows. In section 2 we describe some model-independent aspects of orbifold string-GUTs that are present independently of the construction method used. Some of these may be easily adapted to other left-right symmetric  $(0, 2)$  constructions. We have found that the  $Spin(32)/Z_2$  lattice is a better starting point than  $E_8 \times E_8$  in order to obtain string-GUTs. Due to this we include in section 3 a discussion of some general properties of orbifold models in  $Spin(32)/Z_2$ .

In the following three sections we construct string-GUTs with the three methods of [6]. In the method of continuous Wilson lines used in section 4, the starting point is a  $(0, 2)$  orbifold in which the embedding in the gauge degrees of freedom is totally or partially realized through an automorphism of the gauge lattice (instead of a shift vector). The projection on invariant states forces the left-moving piece of some untwisted fields to be combinations of the usual  $e^{iP \cdot F}$  vertex operators. In particular, Cartan subalgebra states will involve automorphism invariant combinations. The next step is the addition of a continuous Wilson line. Gauge states, including Cartan generators, are eliminated from the massless spectrum and the rank is reduced, often leaving behind a subgroup at higher level.

The second method is described in section 5. This involves the modding by a permutation of identical gauge factors. The starting point is a level  $k = 1$  model in which the orbifold twist is embedded in the gauge degrees of freedom through a shift in the gauge lattice. The orbifold and the shift are chosen so that the observable gauge group has repeated factors such as  $SU(5) \times SU(5)$  or  $SO(10) \times SO(10)$ . More generally, gauge groups of the form  $G_{GUT} \times \hat{G}$  with  $G_{GUT} \subseteq \hat{G}$  are also used. The next step is to add a discrete Wilson line realized as a permutation of the repeated group factors. The projection on invariant states applied to the gauge fields requires forming symmetric combinations of the generators of the two groups, leading to a surviving diagonal group realized at higher level.

In the method of flat directions discussed in section 6, the starting point is also a level  $k = 1$  model. In particular cases where there are scalar field directions flat to all orders, the original gauge symmetry can be spontaneously broken to a subgroup which is realized at  $k > 1$ . Unlike the other two methods that are stringy in nature, this is field-theoretical since the flat directions are analyzed in terms of the effective  $N = 1$  supersymmetric Lagrangian. A typical base model would contain the gauge group  $SU(5) \times SU(5)$  together with massless fields transforming as  $(5, \bar{5})$  and  $(\bar{5}, 5)$ . If there is a flat direction in which these fields acquire an appropriate Vev, the gauge group is broken to a diagonal  $SU(5)$  realized at level  $k = 2$ . Likewise,  $(10, 10)$  multiplets can break  $SO(10) \times SO(10)$  to the diagonal  $SO(10)$  at level  $k = 2$ .

In section 7 we discuss different phenomenological aspects of the class of string-GUTs encountered including a discussion of the generic features of the  $SO(10)$  GUTs found, the structure of the GUT-Higgs potential, the doublet-triplet splitting problem and other general features. We present some final comments and an outlook

in section 8. While constructing the specific string-GUTs we had to resolve several subtleties concerning the generalized GSO projectors in orbifolds with Wilson lines as well as other technical issues which are discussed in the appendix.

## 2 General properties of higher level orbifold GUTs

In this section we discuss general properties of higher level [16] orbifold constructions. Some of these properties are actually model-independent and will also apply to other types of 4-D strings. Our starting point is the ten-dimensional heterotic string in which the gauge degrees of freedom arise from the extra 16 left-moving coordinates  $F_L$  compactified on a torus with  $E_8 \times E_8$  or  $Spin(32)/Z_2$  lattice. The observable gauge group  $E_8 \times E_8$  or  $SO(32)$  corresponds to an affine Lie algebra at level  $k = 1$  [16]. A further process of compactification and twisting will generically lead to a 4-D model with gauge group  $G_1 \times G_2 \times \dots$  in which each factor is associated with an algebra at level  $k_i$ . We recall that for a non-Abelian algebra,  $k$  must be a positive integer whereas for a  $U(1)$  factor,  $k$  is not really a level but a real positive normalization constant. The possible non-Abelian levels are constrained [5, 6, 7] by the condition that the total contribution  $c_G$  of the gauge sector to the (left) central charge satisfies  $c_G \leq 22$  (or  $c_G \leq 16$  if there are no enhanced gauge symmetries). More precisely,

$$c_G = \sum_i c_i = \sum_i \frac{k_i \dim G_i}{k_i + \rho_i} \leq 22 \quad (1)$$

where  $\dim G_i$  and  $\rho_i$  are respectively the dimension and the dual Coxeter number of  $G_i$ . In particular,  $\rho = N$  for  $SU(N)$ ,  $\rho = 2(N - 1)$  for  $SO(2N)$  and  $\rho = 12, 18, 30$  for  $E_{6,7,8}$ . A  $U(1)$  factor contributes 1 to the sum. Condition (1) immediately gives useful information on the possible levels of interesting GUT groups. For example,  $SO(10)$  or  $E_6$  can at most be realized at levels 7 and 4 respectively (4 and 3 if there are no enhanced symmetries). For simply-laced groups  $c_i = \text{rank} G_i$  when  $k_i = 1$ . Since  $c_i$  increases with the level we can also conclude that higher level models necessarily have lower rank.

A second important constraint concerns the possible particles which may appear in the string spectrum, both massless and massive. At level  $k$ , the allowed unitary highest-weight representations must satisfy the condition:

$$\sum_{j=1}^{\text{rank} G} n_j m_j \leq k \quad (2)$$

where  $n_j$  are the Dynkin labels of the highest weight of the representation of  $G$ , and  $m_j$  are positive integers ( $\leq 6$ ) known for every simple lie group. In particular, for  $SU(N)$ ,  $m_j = 1$ , so that for  $k = 1$  the only allowed representations are those with Dynkin levels  $(1, 0, \dots)$ , i.e., the fundamental and completely antisymmetric representations. For  $SO(2N)$  only the vector and spinor representations are allowed at  $k = 1$ . Since adjoint scalars do not appear in the spectrum, the possibility of constructing GUT-like string models at level  $k = 1$  is ruled out.

There are stronger constraints [6, 7] on the possible particles which could be present in the *massless* spectrum. Naively speaking, in string theory the more quantum numbers a particle has, the less likely for it to be massless. This is a very important property of string theories which is often not sufficiently emphasized. Let

$( v_1 ,  v_2 ,  v_3 )$	$E_0$	<u>24</u>	$(5, \bar{5}), \underline{45}$	$(10, 10), \underline{54}$
$(0, 0, 0)$	0	$\checkmark$	$\checkmark$	$\checkmark$
$(1/3, 1/3, 2/3)$	1/3			
$(1/2, 1/4, 1/4)$	5/16			
$(1/3, 1/6, 1/6)$	1/4	$\checkmark$		
$(1/2, 1/3, 1/6)$	11/36			
$(3/7, 2/7, 1/7)$	2/7	$\checkmark$		
$(1/2, 1/8, 3/8)$	19/64			
$(1/4, 1/8, 3/8)$	17/64	$\checkmark$		
$(1/3, 1/12, 5/12)$	13/48	$\checkmark$		
$(1/2, 1/12, 5/12)$	41/144	$\checkmark$		
$(0, 1/2, 1/2)$	1/4	$\checkmark$		
$(0, 1/3, 1/3)$	2/9	$\checkmark$		
$(0, 1/4, 1/4)$	3/16	$\checkmark$	$\checkmark$	
$(0, 1/6, 1/6)$	5/36	$\checkmark$	$\checkmark$	

Table 1: Massless GUT-Higgs fields allowed in the different twisted sectors of all Abelian orbifolds.

us now discuss this property in the context of orbifold models. The conclusions may be generalized to other types of 4-D string constructions. The mass formula for the left-movers of a heterotic 4-D string is given by:

$$\frac{1}{8}M_L^2 = N_L + h_{KM} + E_0 - 1 . \quad (3)$$

Here  $N_L$  is the left-moving oscillator number,  $h_{KM}$  is the contribution of the gauge sector to the conformal weight of the particle and  $E_0$  is the contribution of the internal (compactified) sector to the conformal weight.

Let us consider first the case of symmetric  $(0, 2)$  Abelian orbifolds. All Abelian  $Z_N$  and  $Z_N \times Z_M$  orbifolds may be obtained by toroidal compactifications in which the 6 (left and right) compactified dimensions are twisted. There are just 13 possible orbifold twists which can be characterized by a shift  $v = (v_1, v_2, v_3)$ , where  $e^{2i\pi v_i}$  are the three twist eigenvalues in a complex basis. The 13 possible shifts are shown in Table 1. A consistent symmetric orbifold model is obtained by combining different twisted sectors in a modular invariant way. This procedure is well explained in the literature [17, 18]. Let us just mention that a given twist in the table can be present in several different orbifolds. For example, the shift  $(0, 1/6, 1/6)$  appears in the  $Z_{12}$ ,  $Z_3 \times Z_6$ ,  $Z_2 \times Z_6$  and  $Z_6 \times Z_6$  orbifolds.

To each possible twisted sector there corresponds a value for  $E_0$  given by the general formula:

$$E_0 = \sum_{i=1}^3 \frac{1}{2} |v_i| (1 - |v_i|) \quad (4)$$

Notice also that  $E_0 = 0$  for the untwisted sector which is always part of any orbifold model. The value of  $E_0$  is also shown in Table 1. In the case of asymmetric orbifolds, obtaining  $N = 1$  unbroken SUSY allows the freedom of twisting the right-movers while leaving untouched the (compactified) left-movers. In this case one can then have  $E_0 = 0$  even in twisted sectors.

<b>SU(5)</b>	<b>5</b>	<b>10</b>	<b>24</b>	<b>15</b>	<b>40</b>	<b>50</b>
<b>k = 1</b>	2/5	3/5	-	-	-	-
<b>k = 2</b>	12/35	18/35	5/7	28/35	33/35	-
<b>SO(10)</b>	<b>10</b>	<b>16</b>	<b>45</b>	<b>54</b>	<b>120</b>	<b>126</b>
<b>k = 1</b>	1/2	5/8	-	-	-	-
<b>k = 2</b>	9/20	9/16	4/5	1	-	-
<b>SU(4) × SU(2) × SU(2)</b>	<b>(4, 1, 2)</b>	<b>(1, 2, 2)</b>	<b>(6, 1, 1)</b>	<b>(6, 2, 2)</b>	<b>(20, 1, 1)</b>	<b>(1, 3, 3)</b>
<b>k = 1</b>	5/8	1/2	1/2	1	-	-
<b>k = 2</b>	1/2	3/8	5/12	19/24	1	1

Table 2: Conformal weights  $h_{KM}$  for different representations of the unifying groups  $SU(5)$ ,  $SO(10)$  and  $SU(4) \times SU(2) \times SU(2)$ .

Let us go now to the other relevant piece in eq. (3), namely the contribution  $h_{KM}$  of the affine algebra sector to the conformal weight of the particle. If the orbifold twist is just embedded in the gauge degrees of freedom through a shift  $V$ , each non-Abelian factor of the resulting group inherits level  $k_i = 1$  and furthermore  $h_{KM} = (P + V)^2/2$ . More generally, we assume that further action on internal and gauge degrees of freedom leads to factor groups at higher levels. A state in a representation  $(R_1, R_2, \dots)$  will then have

$$h_{KM} = \sum_i \frac{C(R_i)}{k_i + \rho_i} \quad (5)$$

Here  $C(R)$  is the quadratic Casimir of the representation  $R$ .  $C(R)$  may be computed using  $C(R)\dim(R) = T(R)\dim G$ , where  $T(R)$  is the index of  $R$ . Unless otherwise explicitly stated, we use the standard normalization in which  $T = 1/2$  for the  $N$ -dimensional representation of  $SU(N)$  and  $T = 1$  for the vector representation of  $SO(2N)$ . With this normalization, for simply-laced groups the Casimir of the adjoint satisfies  $C(A) = \rho$ . The contribution of a  $U(1)$  factor to the total  $h_{KM}$  is instead given by  $Q^2/k$ , where  $Q$  is the  $U(1)$  charge of the particle and  $k$  is the normalization of the  $U(1)$  generator, abusing a bit it could be called the level of the  $U(1)$  factor. Formula (5) is very powerful because the  $h_{KM}$  of particles can be computed without any detailed knowledge of the given 4-D string. This information is a practical guide in the search for models with some specific particle content.

In this article we are mainly interested in the construction of GUT models with gauge groups  $SU(5)$  and  $SO(10)$  at level  $k = 2$ . As it will become clear, to this end it is sometimes useful to look for models of the form  $SU(5) \times SU(5)$  and  $SO(10) \times SO(10)$  at level  $k = 1$ . The values of  $h_{KM}$  for the lowest dimensional representations of these groups are given in Tables 2 and 3. We have also included the equivalent results for some representations of the  $SO(10)$  subgroup  $SU(4) \times SU(2) \times SU(2)$ . Notice that the values of  $h_{KM}$  given in these tables should be considered as lower bounds on  $h_{KM}$  since in specific models a given representation, e.g. a 24 of  $SU(5)$ , could be charged with respect to other gauge groups in the model, e.g. a  $U(1)$  factor might be present.

Using eq. (3), the values for  $E_0$  in Table 1 and those for  $h_{KM}$  in Tables 2,3, we can learn, for instance, what  $SU(5)$  or  $SO(10)$  representations may appear in any possible twisted sector of any given Abelian orbifold. In the case of these groups

$SU(5) \times SU(5)$	$(5, \bar{5})$	$(10, 5)$	$(10, 10)$
$k = 1$	$4/5$	$1$	$6/5$
$SO(10) \times SO(10)$	$(10, 10)$	$(10, 16)$	$(16, 16)$
$k = 1$	$1$	$9/8$	$10/8$

Table 3: Conformal weights  $h_{KM}$  for different representations of the unifying groups  $SU(5) \times SU(5)$  and  $SO(10) \times SO(10)$  ( $k = 1$ ).

we are interested in knowing which twisted sectors may contain 24-plets or 45 and 54-plets respectively. In the case of  $SU(5) \times SU(5)$  or  $SO(10) \times SO(10)$  we need to find out which sectors may contain  $(5, \bar{5})$ 's or  $(10, 10)$ 's respectively. The answer to these questions is shown in the last three columns of Table 2 and in Table 3. For a 24-plet ( $k = 2$ ) one has  $h_{KM} = 5/7$ ; for both  $(5, \bar{5})$  and  $SO(10)$  45-plets ( $k = 2$ ) one has  $h_{KM} = 4/5$  and, finally, for both  $(10, 10)$  and  $SO(10)$  54-plets ( $k = 2$ ) one has  $h_{KM} = 1$ . From these results we draw the following conclusions:

- i. All representations shown may be present in the untwisted sector of any orbifold.
- ii. 54s of  $SO(10)$  ( $k = 2$ ) and  $(10, 10)$ s of  $SO(10) \times SO(10)$  ( $k = 1$ ) can only be present in the untwisted sector of symmetric orbifolds.
- iii.  $(5, \bar{5})$ s of  $SU(5) \times SU(5)$  ( $k = 1$ ) and 45s of  $SO(10)$  ( $k = 2$ ) may only appear either in the untwisted sector or else in twisted sectors of the type  $v = 1/4(0, 1, 1)$  or  $v = 1/6(0, 1, 1)$ . This is a very restrictive result since Abelian orbifolds containing these shifts are limited. Notice that the order four shift appears only in the orbifolds  $Z_8, Z_{12}, Z_2 \times Z_4$  and  $Z_4 \times Z_4$ . The order six shift is present in  $Z'_{12}, Z_2 \times Z_6, Z_3 \times Z_6$  and  $Z_6 \times Z_6$ .
- iv. 24-plets of  $SU(5)$  can never appear in the twisted sectors of the  $Z_3, Z_4, Z'_6$  and  $Z_8$  orbifolds.

Table 2 gives us also some extra hints. We observe that the 54-plet of  $SO(10)$  and the  $(10, 10)$  of  $SO(10) \times SO(10)$  not only are forced to be in the untwisted sector but have exactly  $h_{KM} = 1$ . Thus they can potentially be associated to untwisted moduli (continuous Wilson lines, in the language of Refs. [19, 20]). This will turn out to be the case in specific orbifold models, as will be shown in section 4.

From the above conclusions it transpires that looking for models with GUT-Higgs fields in the untwisted sector should be the simplest option, since they can always appear in *any* orbifold. This option has another positive aspect in that the multiplicity of a given representation in the untwisted sector is never very large, it is always less or equal than three in practically all orbifolds and is normally equal to one in the case of  $(0, 2)$  models. Proliferation of too many GUT-Higgs multiplets will then be avoided.

In building models with the GUT-Higgs fields in the untwisted sector one is naturally led to work with orbifolds on the  $Spin(32)/Z_2$  lattice as we now explain with a simplified argument. In models based on the  $E_8 \times E_8$  lattice, the matter fields in the untwisted sector are either charged with respect to the first  $E_8$  or with respect

to the second but there are *no* untwisted matter fields which may be charged with respect to both. Thus, if one has a GUT group of the form  $G \times G$ , there will not be untwisted matter fields transforming as  $(R, R)$  because each  $G$  factor necessarily lies in a different  $E_8$ . But this type of matter is in general needed, at least in the first and third methods above, in order to obtain a diagonal GUT with gauge group  $G$  at  $k = 2$ .

Surprisingly enough, although 4-D orbifolds on the  $E_8 \times E_8$  lattice have been exhaustively classified and analysed, we are not aware of any general analysis of 4-D orbifold strings based on the  $Spin(32)/Z_2$  lattice. Since these compactifications have inherently interesting properties we will briefly discuss them in the following section.

### 3 Abelian orbifolds on the $Spin(32)/Z_2$ lattice

Of course, orbifolds are constructed in the same way both on  $E_8 \times E_8$  and on  $Spin(32)/Z_2$ , the only difference being that in the latter case the gauge lattice consists of 16-dimensional vectors of the form

$$\begin{aligned} & (n_1, n_2, \dots, n_{16}) \\ & (n_1 + \frac{1}{2}, n_2 + \frac{1}{2}, \dots, n_{16} + \frac{1}{2}) \end{aligned} \tag{6}$$

with integers  $n_i$  satisfying  $\sum n_i = \text{even}$ . Since the lattice, denoted  $\Lambda_{16}$ , is self-dual, any  $P \in \Lambda_{16}$  has  $P^2 = \text{even}$ . One important practical difference with the  $E_8 \times E_8$  case is that the shorter spinorial weights have  $P^2 = 4$ . Hence,  $SO(2N)$  spinorial representations cannot appear in the untwisted sector. Likewise, exceptional observable groups like  $E_6$  and  $E_7$  are not possible.

The modular invariance constraints on the possible gauge embeddings are the usual ones. If we associate to a  $Z_N$  twist  $v$  a corresponding shift  $V$  in  $\Lambda_{16}$ , modular invariance of the partition function dictates:

$$N (V^2 - v^2) = 0 \text{ mod } 2. \tag{7}$$

Also,  $NV \in \Lambda_{16}$ . In the case of a  $Z_M \times Z_N$  orbifold [21] ( $M \leq N$ ) with twists  $a, b$  realized through gauge shifts  $A, B$  one has:

$$\begin{aligned} M (A^2 - a^2) &= 0 \text{ mod } 2 \\ N (B^2 - b^2) &= 0 \text{ mod } 2 \\ M (A \cdot B - a \cdot b) &= 0 \text{ mod } 2 \end{aligned} \tag{8}$$

Also,  $MA, NB \in \Lambda_{16}$ . In the presence of a discrete Wilson line  $L$ , the effective lattice shift becomes  $V + nL$  with  $n$  depending on the particular element of the space group considered (see Refs. [18, 19, 22] and the Appendix for more details). The embedding of the twist and discrete Wilson lines in the gauge degrees of freedom may also be realized by automorphisms of  $\Lambda_{16}$  instead of shifts. In this case modular invariance restricts the possible automorphisms allowed as will be exemplified in the next section.

In any orbifold there is always a modular invariant lattice shift [17] that corresponds to embedding the orbifold shift into an  $SO(6) \in SO(32)$ . This standard embedding gives  $(2, 2)$  models and basically amounts to setting  $V = v$  in  $Z_N$  and



$A = a$ ,  $B = b$  in  $Z_M \times Z_N$ . Whereas in  $E_8 \times E_8$  the standard embedding leads to  $E_6$  theories which are chiral, in the  $Spin(32)/Z_2$  case the resulting theories have the non-chiral uninteresting gauge group  $SO(26)$ . This is the sole reason why compactifications on  $\Lambda_{16}$  have been essentially ignored in the literature. While this is a sensible attitude towards  $(2, 2)$  compactifications, more general  $(0, 2)$  theories, that are in fact normally the case, deserve more attention. Appropriate embeddings on  $\Lambda_{16}$  do lead to  $(0, 2)$  models that are more suitable in our approach to constructing standard GUTs.

As we said, in the  $E_8 \times E_8$  case we have the practical knowledge of an embedding, the standard one, that is always modular invariant for *any* orbifold and leads to a chiral model. We do not know of an embedding in  $Spin(32)/Z_2$  which is modular invariant for *any orbifold* and leads to a chiral model. However we have found that both for  $Z_N \times Z_M$  and  $Z_N$  orbifolds on  $\Lambda_{16}$  there is a natural embedding which we call the *five-fold standard embedding* that leads to a chiral model and is *almost always* modular invariant. Furthermore, it naturally provides for  $SU(5)$  and  $SO(10)$  unification in the same sense that the usual standard embedding provides for  $E_6$  unification in the  $E_8 \times E_8$  case.

Let us now explain the idea behind this five-fold standard embedding. Consider the  $SO(6)$  tangent group of the compactified space and its subgroup  $SO(2) \times SO(2) \times SO(2)$ . We want to embed the latter in a symmetric way into  $SO(32)$ . This motivates us to consider the subgroup  $SO(10) \times SO(10) \times SO(10) \times U(1)_A$  of  $SO(32)$  and associate:

$$SO(2) \times SO(2) \times SO(2) \in SO(6) \longrightarrow SO(10) \times SO(10) \times SO(10) \in SO(32) \quad (9)$$

To implement the embedding we associate to an  $SO(6)$  shift  $v$  an  $SO(32)$  shift  $V$  as follows:

$$v = \frac{1}{N}(a, b, c) \longrightarrow V = \frac{1}{N}(a, a, a, a, a, b, b, b, b, b, c, c, c, c, c, d) \quad (10)$$

It is now obvious why we call it five-fold standard embedding, it contains five times the standard embedding shift. The last integer  $d$  is associated to the remaining  $U(1)_A$  symmetry and its value is fixed by modular invariance. Also,  $NV$  must belong to  $\Lambda_{16}$ .

We now consider explicitly the case of  $Z_M \times Z_N$  orbifolds. We recall that the possible values of  $M$  are  $M = 2, 3, 4, 6$  and  $N = \alpha M$  for some  $\alpha = 1, 2, 3$ . The five-fold embedding of the shift  $a = \frac{1}{M}(1, 0, -1)$  is then given by

$$A = \frac{1}{M}(1, 1, 1, 1, 1, 0, 0, 0, 0, 0, -1, -1, -1, -1, -1, d_a) \quad (11)$$

A suitable value of  $d_a$  can always ensure the corresponding modular invariance constraint for all  $M$ . A twist of this type leaves a  $N = 2$  unbroken SUSY. The shift  $A$  in (11) implies a gauge group  $SU(10) \times SO(10) \times U(1)^2$ , enhanced to  $SO(20) \times SO(10) \times U(1)^2$  in the  $Z_2$  case. To further reduce to  $N = 1$  supersymmetry one considers the simultaneous shift  $b = \frac{1}{N}(0, 1, -1)$ . Its five-fold embedding is given by

$$B = \frac{1}{N}(0, 0, 0, 0, 0, 1, 1, 1, 1, 1, -1, -1, -1, -1, -1, d_b) \quad (12)$$

It is easy to check that the above shifts  $A$  and  $B$  satisfy the conditions in (8) for all  $M$  and  $N$  by choosing, for example,  $d_a = 4$  and  $d_b = 8$ .

The generic gauge group of the  $Z_M \times Z_N$  five-fold embedding is

$$SU(5) \times SU(5) \times SU(5) \times U(1)^3 \times U(1)_A \quad (13)$$

It is enhanced to  $SO(10)^3 \times U(1)_A$  in  $Z_2 \times Z_2$  and to  $SU(5)^2 \times SO(10) \times U(1)^2 \times U(1)_A$  in  $Z_2 \times Z_4$ . The Abelian factor  $U(1)_A$  is anomalous and in all the cases studied its anomaly is cancelled in the usual way by the 4-D version of the Green-Schwarz mechanism. This will turn out to have important consequences for the one-loop stability of the string vacua that we will be considering.

The untwisted matter content has also some interesting features common to all resulting models. In particular, it contains the multiplets

$$[(5, \bar{5}, 1) + (\bar{5}, 1, 5) + (1, 5, \bar{5})] \\ + 2[(5, 1, 1) + 2(1, 5, 1) + 2(1, 1, 5)] \quad (14)$$

where, depending on the particular model, there may be flippings  $5 \leftrightarrow \bar{5}$  in some  $SU(5)$  factor. In some cases there may also appear *additional* untwisted matter fields such as 10-plets in  $Z_3 \times Z_3$ . The interesting point is that fields with these characteristics will be important in obtaining appropriate GUT-Higgs fields as we will see later on.

In  $Z_N$  orbifolds the same construction essentially applies. The five-fold embedding of  $Z_3, Z_4, Z_6, Z_7$  and  $Z_{12}$  orbifolds is modular invariant with  $d_v = 0$ , whereas  $d_v = 4$  is necessary for  $Z'_6$  and  $Z'_{12}$ . For  $Z_N$  orbifolds of small  $N$  one single shift is not enough to achieve all the breaking down to  $SU(5)^3 \times U(1)^3$ . For example, this group is enhanced to  $SU(15) \times U(1)^2$  in  $Z_3$ . Further addition of Wilson lines would in general be needed to arrive at the smaller group.

There is an exception to the universal validity of the five-fold embedding, for the orbifold of order 8 there is no possible choice of  $d_v$  that renders this embedding modular invariant. In spite of this lack of generality, the five-fold embedding is interesting since it leads in a natural way to gauge groups which are of phenomenological interest. Furthermore, a natural replication of these groups occurs.

We wish to emphasize that there are many other embeddings in  $\Lambda_{16}$  that lead to GUT groups, whether repeated or not. We will exploit several possibilities. In particular, we will also consider embeddings through automorphisms of the gauge lattice.

Unlike automorphisms of the  $E_8$  lattice [23], those of  $\Lambda_{16}$  have not been studied in any detail. We now give a simplified analysis adapted to our future needs. We are mostly interested in automorphisms of order 2 and 4. It is easy to see that two simultaneous sign flips  $F_I \rightarrow -F_I$  or two simultaneous  $\pi/2$  rotations  $F_I \rightarrow F_J$ ,  $F_J \rightarrow -F_I$ , are allowed automorphisms of  $\Lambda_{16}$ . We use these transformations as basic building blocks.

Modular invariance, or equivalently left-right level-matching further restricts the allowed automorphisms. More precisely, for a  $Z_N$  automorphism  $\Theta$  we must have

$$E_\Theta + E_0 - 1 = 0 \mod \frac{1}{N} \quad (15)$$

where  $E_\Theta$  is the vacuum energy shift due to the  $\Theta$ -rotated  $F$ -coordinates. Notice that  $E_\Theta$  can be computed by a formula similar to (4). For instance, we find  $E_\Theta = p/8$  for a  $Z_2$  automorphism in which  $2p$  coordinates change sign. Such gauge action can

accompany an internal shift  $v = (\frac{1}{2}, 0, -\frac{1}{2})$  with  $E_0 = 1/4$  provided  $p = 2, 6$ . Likewise, for a  $Z_4$  automorphism in which  $2r$  coordinates change sign and  $4s$  coordinates are rotated by  $\pi/2$ , we find  $E_\Theta = (3s + 2r)/16$ . This automorphism can then act as embedding of the shift  $v = (\frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$  with  $E_0 = 5/16$  provided  $s = 1$ ,  $r = 0, 2, 4, 6$  or  $s = 3$ ,  $r = 1$ .

Lattice shifts equivalent to a given automorphism can also be determined. For example,

$$\Theta(F_1, F_2, \dots, F_{16}) = (-F_1, -F_2, -F_3, -F_4, F_5, \dots, F_{16}) \quad (16)$$

is equivalent to

$$V = (\frac{1}{2}, \frac{1}{2}, 0, \dots, 0) \quad (17)$$

Similarly,

$$\Theta(F_1, F_2, \dots, F_{16}) = (F_2, -F_1, F_4, -F_3, F_5, \dots, F_{16}) \quad (18)$$

is equivalent to

$$V = (\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 0, \dots, 0) \quad (19)$$

The above results can be verified by comparing the spectrum in the two formulations.

## 4 GUTs from continuous Wilson lines: the GUT-Higgs as a string modulus

The method of continuous Wilson lines was first introduced in Refs. [19, 18] as a stringy procedure to reduce the rank of the gauge group in 4-D orbifold models. Its relationship with the stringy Higgs mechanism was analyzed in Refs. [20, 24] and recently [25] a classification of the untwisted moduli space in the case of  $E_8 \times E_8$  was worked out for the models obtained using this method. In Ref. [6] it was explicitly shown how under some circumstances it also leads to higher level orbifold models. Below we review its basic features in the case of  $Z_N$  orbifolds.

The method relies on the non-Abelian embedding of the orbifold space group with elements  $(\theta, n_i e_i)$ , where  $\theta$  is the orbifold twist and  $e_i$  is an internal six-dimensional lattice vector. The associated action is given by  $(\Theta, n_i L_i)$ , where  $\Theta$  is an order  $N$  automorphism and  $L_i$  is a translation of the gauge lattice. In the absence of Wilson lines  $L_i$ , the action of  $\Theta$  can be described by an equivalent shift  $V$ . In the presence of  $L_i$ , the embedding is non-Abelian when  $\Theta L_i$  does not give back  $L_i$  up to lattice vectors. In the following we consider the case of  $L_i$  completely rotated by  $\Theta$  so that the condition  $(\Theta, n_i L_i)^N = (1, 0)$  is automatically fulfilled. This implies that the Wilson lines  $L_i$  are not quantized but may take arbitrary real values and be continuously varied.

When embedding by automorphisms, not all Cartan gauge currents are given by combinations of derivatives  $\partial F_I$  since the lattice coordinates  $F_I$  are generically rotated by  $\Theta$  and the unbroken gauge currents must be invariant under  $\Theta$ . The Cartan sub-algebra, as well as the step currents, now arise from  $\Theta$  invariant orbits of the  $e^{iP \cdot F}$  operators of the form

$$|P\rangle + |\Theta P\rangle + \dots + |\Theta^{N-1} P\rangle \quad (20)$$

where  $|P\rangle \equiv e^{iP \cdot F}$  and  $P^2 = 2$ . On the other hand, untwisted matter states will involve combinations of the form

$$|P\rangle + \delta|\Theta P\rangle + \dots + \delta^{N-1}|\Theta^{N-1}P\rangle \quad (21)$$

that acquire a phase  $\delta$ ,  $\delta^N = 1$ , under  $\Theta$ . This phase compensates for the transformation of the right-moving piece of the full vertex. Combinations of  $\partial F_I$  states can also give rise to untwisted matter.

After the continuous Wilson lines are turned on, states not satisfying  $P \cdot L_i = \text{int}$  drop out from the spectrum. This projection kills some Cartan generators thus forcing a reduction of the rank of the gauge group. This is a necessary condition to get a residual algebra realized at higher level.

Concerning the twisted sectors of the orbifold, the left-handed mass formula now becomes

$$\frac{1}{8}M_L^2 = \frac{1}{2}(P_T + n_i L_{iT})^2 + N_L + E_0 + E_\Theta - 1 \quad (22)$$

where  $E_\Theta$  is the vacuum energy shift due to  $\Theta$ .  $P_T$  and  $L_{iT}$  are the components of  $P$  and  $L_i$  which are left unrotated by  $\Theta$ . Notice that there is no winding in the rotated directions. Also,  $N_L$  can now take fractional values both due to the rotated  $F$ -coordinates and the compactified dimensions. States in twisted sectors organize into representations whose dimensionality depends on the degeneracy factor

$$D = \sqrt{\frac{\det'(1 - \Theta)}{|I^*/I|}} \quad (23)$$

where  $\det'$  is evaluated in the rotated piece of the lattice.  $I$  is the sub-lattice left invariant by  $\Theta$ , and  $|I^*/I|$  is the index of its dual  $I^*$  on  $I$ . This factor is similar to that appearing in asymmetric orbifolds [26] because the gauge twisting  $\Theta$  is asymmetric in nature. Since  $\Theta$  does not affect any right-movers,  $D$  is roughly speaking the square root of the number of points fixed under  $\Theta$ . These fixed points in general belong to the unbroken group weight-lattice and therefore are non-trivially charged. This means that the degeneracy factor corresponds to some (reducible) representation of the unbroken gauge group [26].

Before building explicit models let us comment that since the gauge piece is a complicated linear combination, it is often difficult to quickly identify the representation and quantum numbers of a given massless state. To this purpose it proves convenient to use a parallel description of the original orbifold, without Wilson lines  $L_i$ , in terms of the shift  $V$  equivalent to the action of  $\Theta$ . In this way, gauge quantum numbers can be more easily determined.

To illustrate the continuous Wilson line method we are going to build an  $SO(10)$  GUT realized at level  $k = 2$ . We will consider the simplest symmetric orbifold with order 2 symmetries, namely,  $Z_2 \times Z_2$ . The internal six-dimensional twists  $\theta$  and  $\omega$  are respectively realized by the order two automorphisms  $\Theta$  and  $\Omega$  defined by :

$$\begin{aligned} \Theta(F_1, F_2, \dots, F_{16}) &= (-F_1, -F_2, \dots, -F_{10}, -F_{11}, -F_{12}, F_{13}, F_{14}, F_{15}, F_{16}) \\ \Omega(F_1, F_2, \dots, F_{16}) &= (-F_1, -F_2, \dots, -F_{10}, F_{11}, F_{12}, -F_{13}, -F_{14}, F_{15}, F_{16}) \end{aligned} \quad (24)$$

Notice that  $\Theta\Omega$  is another  $Z_2$  allowed automorphism. The unbroken gauge currents correspond to states  $|P\rangle$  with  $P$  invariant plus the oscillators  $\partial F_{15}, \partial F_{16}$ . Also, from non-invariant  $P$ 's we can form orbits invariant under both  $\Theta$  and  $\Omega$ . Altogether we

find 200 currents that can be organized into an  $SO(10) \times SO(18) \times U(1)^2$  algebra realized at level  $k = 1$ .

The untwisted matter includes states transforming as  $(10, 1)$ ,  $(1, 18)$ ,  $(10, 18)$  and singlets  $(1, 1)$ . In the twisted sectors we find matter in the representations  $(16, 1)$ ,  $(\overline{16}, 1)$ ,  $(10, 1)$ ,  $(1, 18)$  and  $(1, 1)$ , in multiplicities according to the projectors in (78). At this stage, the model can be equivalently derived through the shifts

$$\begin{aligned} A &= \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right) \\ B &= \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0\right) \end{aligned} \quad (25)$$

In this formulation all charges can be easily determined.

Next we turn on a Wilson line background  $L$  along, say, the compactified direction  $e_6$ .  $L$  has the form

$$L = (\lambda, \lambda, \lambda, \dots, \lambda, 0, 0, 0, 0, 0, 0) \quad (26)$$

The parameter  $\lambda$  can take any real value since  $L$  is completely rotated by both  $\Theta$  and  $\Omega$ . The gauge group is broken to  $SO(10) \times SO(8) \times U(1)^2$ . The associated currents are

$$\begin{aligned} &\textbf{SO}(10) \\ &|\underline{+1, -1, 0, 0, \dots, 0}, 0, 0, 0, 0, 0, 0\rangle + |\underline{-1, +1, 0, 0, \dots, 0}, 0, 0, 0, 0, 0, 0\rangle \\ &\textbf{SO}(8) \\ &|0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \pm 1, \pm 1\rangle \\ &\partial F_{15}, \partial F_{16} \\ &|0, \dots, 0, 0, 0, \underline{+1, 0, \pm 1, 0}\rangle + |0, \dots, 0, 0, 0, \underline{-1, 0, \pm 1, 0}\rangle \\ &|0, \dots, 0, \underline{+1, 0, 0, 0, \pm 1, 0}\rangle + |0, \dots, 0, \underline{-1, 0, 0, 0, \pm 1, 0}\rangle \\ &[|0, \dots, 0, \underline{+1, 0, +1, 0, 0, 0}\rangle + |0, \dots, 0, \underline{-1, 0, +1, 0, 0, 0}\rangle + \\ &|0, \dots, 0, \underline{+1, 0, -1, 0, 0, 0}\rangle + |0, \dots, 0, \underline{-1, 0, -1, 0, 0, 0}\rangle] \\ &[|0, \dots, 0, 0, 0, +1, +1, 0, 0\rangle + |0, \dots, 0, 0, 0, -1, -1, 0, 0\rangle + \\ &|0, \dots, 0, 0, 0, +1, -1, 0, 0\rangle + |0, \dots, 0, 0, 0, -1, +1, 0, 0\rangle] \\ &[|0, \dots, 0, +1, +1, 0, 0, 0, 0\rangle + |0, \dots, 0, -1, -1, 0, 0, 0, 0\rangle + \\ &|0, \dots, 0, +1, -1, 0, 0, 0, 0\rangle + |0, \dots, 0, -1, +1, 0, 0, 0, 0\rangle] \\ &\textbf{U}(1)^2 \\ &[|0, \dots, 0, 0, 0, +1, +1, 0, 0\rangle + |0, \dots, 0, 0, 0, -1, -1, 0, 0\rangle - \\ &|0, \dots, 0, 0, 0, +1, -1, 0, 0\rangle - |0, \dots, 0, 0, 0, -1, +1, 0, 0\rangle] \\ &[|0, \dots, 0, +1, +1, 0, 0, 0, 0\rangle + |0, \dots, 0, -1, -1, 0, 0, 0, 0\rangle - \\ &|0, \dots, 0, +1, -1, 0, 0, 0, 0\rangle - |0, \dots, 0, -1, +1, 0, 0, 0, 0\rangle] \end{aligned} \quad (27)$$

where underlining means that all possible permutations must be properly considered. These are the only states simultaneously invariant under  $\Theta$  and  $\Omega$  involving only momenta satisfying  $P \cdot L = \text{int}$ . The above states can be organized into currents by checking the operator product expansions (OPEs) that reflect their corresponding algebras [16]. Notice that the  $SO(10)$  states are orthogonal to those of  $SO(8)$  and  $U(1)^2$ , i.e. their mixed OPEs are regular. Notice also that the  $U(1)$  combinations are

chosen so that they are orthogonal to  $SO(8)$ . The  $SO(8)$  group is realized at level one, since it contains the level one subgroup  $SO(4)$  untouched from the beginning. On the other hand the  $SO(10)$  algebra is realized at level two as can be verified directly from the OPEs and indirectly in other ways explained below.

In the untwisted sectors  $U_1, U_2$  and  $U_3$ , the corresponding left-moving vertices transform under  $(\Theta, \Omega)$  with eigenvalues  $(-1, 1), (1, -1)$  and  $(-1, -1)$  respectively. The momenta involved must also satisfy  $P \cdot L = \text{int}$ . In sectors  $U_1$  and  $U_2$  there are matter fields transforming as  $(1, 8)$  and with different  $U(1)$  charges. In the  $U_3$  sector we find the states

$$\begin{aligned} & \partial F_I, \quad I = 1, \dots, 10 \\ & |\underline{+1, -1, 0, \dots, 0, 0, 0, 0, 0, 0}\rangle - |\underline{-1, +1, 0, \dots, 0, 0, 0, 0, 0, 0}\rangle \end{aligned} \quad (28)$$

These states have no  $U(1)^2$  charges and belong to a  $(54, 1) + (1, 1)$  representation of  $SO(10) \times SO(8)$ . Checking the structure of the 54 of  $SO(10)$  from OPEs is cumbersome. Fortunately, there is a simpler argument to support this fact. Since the orbit states must have  $h_{KM} = 1$  and they are neutral under  $U(1)^2$  and  $SO(8)$ , they can only belong to a 54 that precisely has  $h = 1$  at  $k = 2$  as shown in Table 1. In  $U_3$  we also find

$$\begin{aligned} & [|0, \dots, 0, \underline{+1, 0, +1, 0, 0, 0}\rangle - (0, \dots, 0, \underline{-1, 0, +1, 0, 0, 0}) - \\ & |0, \dots, 0, \underline{+1, 0, -1, 0, 0, 0}\rangle + |0, \dots, 0, \underline{-1, 0, -1, 0, 0, 0}\rangle] \end{aligned} \quad (29)$$

These are four singlets, charged under the  $U(1)$ s only.

Let us now examine the twisted sectors. Consider first the sector twisted by  $\theta$  and the automorphism  $\Theta$ . The left-handed mass formula is given by eq. (22). Since, in this case  $E_0 + E_\Theta = 1$  we must have  $N_L = 0$  and also  $P_T = 0$  because  $L_T = 0$ . The quantum numbers of the massless states are then essentially given by the degeneracy of the vacuum. The invariant lattice  $I$  is the root lattice of  $SO(8)$ . Its dual is the weight lattice that has four conjugacy classes. Hence,  $|I^*/I| = 4$ . Substituting these value together with  $\det'(1 - \Theta) = 2^{12}$  in eq. (23) we find  $D_\Theta = 32$ . The Wilson line  $L$  merely shifts the position of the fixed points but does not affect the counting.

The value of  $D_\Theta$  suggests that this  $\theta$  sector contains a  $(16, 1) + (\overline{16}, 1)$ . This guess is confirmed by analyzing the equivalent model in terms of shifts instead of automorphisms, *before* adding the Wilson line. In the shift formulation we easily verify that the  $\theta$  sector contains those multiplets. The quantum numbers must be the same for the equivalent model obtained through automorphisms. Moreover, they must be the same in the  $k = 2$  model that is continuously connected by varying the Wilson line. The number of the  $(16, 1)$  and  $(\overline{16}, 1)$  multiplets depends on the specific form of the  $Z_2 \times Z_2$  rotations  $\theta$  and  $\omega$  as explained in the Appendix. With the choice leading to the multiplicity factor in eq. (75) we obtain three  $SO(10)$  generations plus one antigeneration. The  $\omega$  sector also gives three  $(16, 1)$  and one  $(\overline{16}, 1)$  with different  $U(1)$  charges. In the  $\theta\omega$  sector we obtain states transforming as  $(10, 1)$ ,  $(1, 8)$  and singlets.

Altogether the spectrum of this GUT model is given in Table 4. The charge  $Q$  is non-anomalous whereas  $Q_A$  is anomalous. The gravitational, cubic and mixed gauge anomalies of  $Q_A$  must be in the correct ratios in order to be cancelled by the 4-D version of the Green-Schwarz mechanism [27]. In particular, the mixed anomalies

<i>Sector</i>	$SO(10) \times SO(8)$	$Q$	$Q_A$
$U_1$	(1,8)	1/2	1/2
	(1,8)	-1/2	-1/2
$U_2$	(1,8)	-1/2	1/2
	(1,8)	1/2	-1/2
$U_3$	(54,1)	0	0
	(1,1)	0	0
	(1,1)	0	1
	(1,1)	1	0
	(1,1)	-1	0
	(1,1)	0	-1
$\theta$	3(16, 1)	1/4	1/4
	( $\overline{16}$ , 1)	-1/4	-1/4
$\omega$	3(16, 1)	-1/4	1/4
	( $\overline{16}$ , 1)	1/4	-1/4
$\theta\omega$	4(10, 1)	0	1/2
	4(10, 1)	0	-1/2
	3(1, 8)	0	1/2
	(1, 8)	0	-1/2
	8(1, 1)	1/2	0
	8(1, 1)	-1/2	0

Table 4: Particle content and charges of Example 1.

of  $Q_A$  with  $SO(10)$  and  $SO(8)$  should be in the same ratio as the levels  $k_{10}/k_8 = 2$ . We find  $TrQ_A/TrQ_A^3 = 24/3$ ;  $\mathcal{B}_8/TrQ_A^3 = 1/3$  and  $\mathcal{B}_{10}/TrQ_A^3 = 2/3$ , where  $\mathcal{B}$  is the mixed anomaly coefficient. These expected results furnish a consistency check of our construction.

We now wish to discuss an important feature of the GUT Higgs and its singlet partner appearing in the  $U_3$  sector. In the 0-picture the full emission vertex operator for the singlet has the form

$$\partial X_3 \otimes \sum_{I=1}^{10} \partial F_I \quad (30)$$

A Vev for this field precisely corresponds to the Wilson line background  $L$  in eq. (26). The fact that this background may be varied continuously means that this singlet is a *string modulus*, a chiral field whose scalar potential is flat to all orders. Indeed, using the discrete  $Z_2$  R-symmetries of the right-handed sector, it can be proven that its self-interactions vanish identically.

The GUT Higgs contains the other 9 linear combinations of  $\partial F_I$ . These give the diagonal elements of the symmetric traceless matrix chosen to represent the 54-plet. the associated vertex operator is

$$\partial X_3 \otimes \sum_{I=1}^{10} c_I \partial F_I \quad ; \quad c_I \in \mathbf{R}, \quad \sum_I c_I = 0 . \quad (31)$$

Vevs for these nine components of the 54 would correspond to the presence of more general Wilson backgrounds of the form  $L = (\lambda_1, \lambda_2, \dots, \lambda_{10}, 0, 0, 0, 0, 0)$  with

$\sum_{I=1}^{10} \lambda_I = 0$ . These more general backgrounds break the symmetry further to some  $SO(10)$  subgroup like  $SU(4) \times SU(2)_L \times SU(2)_R$ . The fact that these other nine modes may be continuously varied means that they are also string moduli or, more generally, that the 54-plet of  $SO(10)$  in this model is itself a string modulus! We find that this property of the GUT-Higgs behaving as a string modulus, on equal footing with the compactifying moduli  $T_i$ , is very remarkable.

We have constructed with simple methods a level  $k = 2$   $SO(10)$  GUT model with a single GUT-Higgs transforming as a 54. This model has other interesting properties, particularly in the couplings of the Higgs sector as well as in the one-loop Fayet-Iliopoulos (see section 7).

The example summarized in Table 4 belongs to a whole class of models obtained through continuous Wilson lines. A general characteristic is that they are  $SO(10)$  models in which the GUT Higgs is a 54 multiplet. Moreover, there is only one such GUT Higgs coming from the untwisted sector and behaving like a string modulus. On the other hand, the rest of the particle content is model dependent. This includes the number of generations, existence of Higgses 10s,  $(16 + \overline{16})$ s, hidden gauge group, etc.. For instance, the number of generations can be changed by adding discrete Wilson lines to the original orbifold. There are no  $SO(10)$  models in this class with 45s of Higgses instead of 54s. Although our search has been far from complete, we have not found  $SU(5)$  models in this class. For reference we will now give two more examples of this class skipping the details. They could guide the reader in looking for different models.

Our second example is also based on the  $Z_2 \times Z_2$  orbifold but this time one of the  $Z_2$ s is realized by a reflection and the other by a shift. The actual embedding is

$$\begin{aligned} \theta : \quad \Theta(F_1, F_2, \dots, F_{16}) &= (-F_1, -F_2, \dots, -F_{11}, -F_{12}, F_{13}, F_{14}, F_{15}, F_{16}) \\ \omega : \quad B &= (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \end{aligned} \quad (32)$$

At this stage the gauge symmetry is  $SO(10)^3 \times U(1)_A$ .

After turning on a Wilson line of the form in eq. (26), the gauge group breaks to  $SO(10) \times SO(10) \times U(1)_A$  with the first  $SO(10)$  realized at level  $k = 2$ . There are no matter fields in the  $U_2$  and  $U_3$  sectors whereas  $U_1$  contains  $(54 + 1, 1) + 2(1, 10)$ .

In both  $\theta$  and  $\omega$  sectors there are 3 copies of  $(16, 1)$  and one  $(\overline{16}, 1)$ . In the  $\theta\omega$  sector there are instead  $3(1, \overline{16})$  and one  $(16, 1)$ . The initial level one orbifold in this example may equivalently be constructed through the five-fold embedding discussed in section 3. This model is further discussed in section 6 where it is constructed using a different method.

Our third example is based in the symmetric  $Z_4$  orbifold. The single generator  $\theta$  with shift  $v = (\frac{1}{4}, \frac{1}{4} - \frac{1}{2})$  is realized through the automorphism given by

$$v : \Theta(F_1, F_2, \dots, F_{16}) = (F_2, -F_1, F_4, -F_3, -F_5, -F_6, \dots, -F_{15}, -F_{16}) \quad (33)$$

After adding a Wilson-line of the form  $L = (0, \dots, 0, \lambda, \lambda, \lambda, \lambda, \lambda, \lambda)$ , we are left with gauge group  $SO(10) \times SU(2)^5 \times U(1)$  with the  $SO(10)$  realized at  $k = 2$ . This model again has a 54 in the untwisted sector, no massless states in the  $\theta$ -twisted sector and 10s plus hidden matter in the  $\theta^2$  sector.



## 5 Constructing level two string models through permutation modding

In this second method the basic observation is that, when two identical gauge factors  $G \times G$  belonging to a starting level  $k = 1$  model are exchanged, the diagonal group  $G_D$  at level  $k = 2$  emerges as the survivor of the projection over permutation invariant states [6]. This general idea may be implemented in orbifold constructions, in essentially two different ways :

i. The order-two permutation is associated to one of the twists defining the orbifold. This means that in a  $Z_2 \times Z_N$  orbifold,  $Z_2$  is embedded through the permutation  $\Pi$  of two identical gauge factors. The  $Z_N$  action is realized in the usual way, through a shift  $V$  in the lattice  $\Lambda_{16}$ . Since we are dealing with Abelian orbifolds, the two operations must commute. This implies the constraint

$$V = \Pi V \mod \Lambda_{16} \quad (34)$$

where  $\Pi V$  is the shift obtained upon permutation. This turns out to be a very strong requirement. A more general situation in  $Z_M \times Z_N$ , with  $M$  even, may be imagined by embedding the  $Z_M$  twist as an order two permutation plus a shift in the  $\Lambda_{16}$  lattice. Nevertheless, eq. (34) must still be satisfied.

Additional requirements come from modular invariance (level-matching) which essentially limit the number of pairs of coordinates which may be permuted. This is discussed below.

ii. The order two permutation  $\Pi$  mimics the effect of a *quantized* Wilson line in the orbifold. Since this Wilson line has order two, the original  $Z_N$  or  $Z_N \times Z_M$  orbifold must be of even order. Thus  $Z_3$ ,  $Z_3 \times Z_3$  and  $Z_7$  cannot be used for this purpose.

Consistently embedding the space group into the gauge degrees of freedom imposes again severe constraints. Interestingly enough, these constraints depend on the way in which the  $Z_N$  action is realized on the six dimensional lattice. In order to exemplify this point, let us consider a  $Z_N$  orbifold, defined through a given twist  $\theta$  with associated gauge lattice shift  $V$ . The corresponding space-gauge group twisting element is denoted  $(\theta, 0|1, V)$ . We also add a discrete Wilson line along, say, the compactifying lattice vector  $e_1$ . The associated group element  $(1, e_1|\Pi, W)$  implements a shift  $e_1$  in the compactifying lattice and simultaneously acts as permutation  $\Pi$  plus a shift  $W$  in the gauge lattice.

The product element  $(1, e_1|\Pi, W)(\theta, 0|1, V) = (\theta, e_1|\Pi, \Pi V + W)$ , must belong to the space-gauge twisting group. By applying this element  $N = 2j$  times we get

$$\begin{aligned} & (\theta, e_1|\Pi, \Pi V + W)^N = \\ & (\theta^N, e_1 + \theta e_1 + \dots + \theta^{N-1} e_1 | \Pi^N, \Pi V + W + \Pi(\Pi V + W) + \dots + \Pi^{N-1}(\Pi V + W)) = \\ & (1, 0|1, j[\Pi V + W] + j[V + \Pi W]) \end{aligned}$$

For the embedding to be a consistent homomorphism of the space group into the gauge degrees of freedom, the above element should be trivial. Therefore,

$$j[\Pi(V + W) + (V + W)] \in \Lambda_{16} \quad (35)$$

This is a necessary constraint, but depending on the compactifying lattice there could even be additional ones. Let us consider, as an example, the  $Z_4$  orbifold defined

by the eigenvalues  $1/4(1, 1, -2)$ . On a  $SU(4) \times SU(4)$  lattice, above equation with  $j = 2$  is also sufficient. In particular, this means that order four Wilson lines are admitted. For the cubic  $SO(4)^3$  lattice there is another constraint due to the relation

$$e_1 + \theta^2 e_1 = 0 \quad (36)$$

that implies

$$\left. \begin{array}{l} \Pi W + W \\ 2[\Pi V + V] \end{array} \right\} \in \Lambda_{16} \quad (37)$$

Only order two Wilson lines are allowed in this case. Moreover, if we chose to associate the permutation Wilson line to the third  $SO(4)$  lattice, the even more severe constraint equation (34) is found.

From this example we learn that the constraints, and therefore the model building possibilities, coming from the embedding of the space group into the gauge degrees of freedom, crucially depend on the compactifying lattice chosen. This is due to the existence of relations among twist and lattice vectors. A similar situation arises when, in a given lattice, there exist different inequivalent twists that can realize the orbifold action as happens in  $Z_2 \times Z_2$  examples in Appendix.

The construction of type **i.** and **ii.** models follows the usual rules of orbifold model building. Some distinguishing features appear in those twisted sectors in which the twist in the compactified dimensions is accompanied by a permutation in the gauge degrees of freedom. We will now discuss this kind of sectors and provide some examples to illustrate the whole procedure.

Consider an initial  $k = 1$  model including a group  $G \times G$ , with  $G$  of rank  $R$  ( $2R \leq 16$ ). Denote the Cartan generators of the first (second)  $G$  factor by  $\partial X (\partial Y)$  and the remaining (up to 16) by  $\partial Z$ . In the twisted sectors, taking into account the permutation modding, we have the following boundary conditions for the three types of gauge coordinates

$$\begin{aligned} X(u + \pi) &= Y(u) + \pi P_1 + \pi V_1 \\ Y(u + \pi) &= X(u) + \pi P_2 + \pi V_2 \\ Z(u + \pi) &= Z(u) + \pi P_3 + \pi V_3 \end{aligned} \quad (38)$$

where  $u = \sigma - \tau$  is the left-handed world-sheet variable,  $P_i$  are components of vectors  $P \in \Lambda_{16}$  and  $V_i$  are components of a shift which might be present in the specific twisted sector considered. We can write mode expansions for the coordinates  $X$  and  $Y$  corresponding to the two gauge factors,

$$\begin{aligned} X(u) &= X_0 + M_1 u + \frac{i}{2} \sum \frac{x_r}{r} e^{-2iru} \\ Y(u) &= Y_0 + M_2 u + \frac{i}{2} \sum \frac{y_r}{r} e^{-2iru} \end{aligned} \quad (39)$$

where  $M_1$  and  $M_2$  are the quantized momenta. When boundary conditions (38) are imposed it follows that

$$M_1 = M_2 = M = \frac{(P + V)}{2} \quad (40)$$

$$x_r = e^{2i\pi r} y_r ; \quad y_r = e^{2i\pi r} x_r \quad (41)$$

The second conditions in this equation indicate that  $r = m$  or  $r = m + 1/2$ . Therefore, gauge oscillator numbers are either integer or semi-integer. Permutation modding contributes to the vacuum energy by increasing it by  $R/16$ , where  $R$  is the

number of permuted pairs of coordinates. Altogether, we conclude that the left-handed mass formula in a permuted twisted sector is given by

$$\frac{1}{8}m_L^2 = N_L + \frac{(P_\pi + V_\pi)^2}{4} + \frac{(P_3 + V_3)^2}{2} + E_0 + \frac{R}{16} - 1 \quad (42)$$

where  $P_\pi + V_\pi = P_1 + P_2 + V_1 + V_2 = 2M$ . Here  $N_L$  stands for the oscillator numbers from both the compactifying twist and the permutation modding and  $E_0$  is the vacuum energy from the twist, as given in Table 1. This is what we essentially need to compute the massless states in these permuted sectors.

As mentioned above, for the permutation method to really give rise to a level 2 model, the permutation modding must be performed among gauge coordinates  $X$  and  $Y$  which *do not* belong to the same gauge group. Instead they must correspond to gauge factors  $G_1$  and  $G_2$  well differentiated.

Since we want to obtain GUT gauge groups such as  $SU(5)$ ,  $SO(10)$  and even  $E_6$ , a natural possibility is to embed each of the two identical gauge groups into a different  $E_8$  factor of the  $E_8 \times E_8$  heterotic string and then do a permutation modding of the coordinates of both  $E_8$ s. Let us assume that the permutation is associated to a twist, as considered in **i.**. Therefore eq. (34) must be verified, leading to  $E_8 \leftrightarrow E'_8$  symmetric shifts of the form

$$V = \frac{1}{N}(d_1, d_2, \dots, d_8) \otimes \frac{1}{N}(d_1, d_2, \dots, d_8) \quad (43)$$

associated to the 6-dimensional twist  $v = \frac{1}{N}(a, b, c)$ . It is easy to prove, however, that there are no such symmetric shifts which are modular invariant for any *even order*  $Z_N$  orbifold. Indeed, the modular invariance condition eq. (7) implies

$$2(d_1^2 + \dots + d_8^2) - (a^2 + b^2 + c^2) = 0 \mod 2N \quad (44)$$

but the first term is necessarily  $0 \mod 4$ , whereas for all Abelian orbifolds,  $(a^2 + b^2 + c^2)$  is always  $2 \mod 4$ . Since  $N$  is even by hypothesis, we conclude that (44) cannot be fulfilled.

The second type of construction leads to similar conclusions if eq. (35) must be satisfied. Thus we see that the permutation modding mechanism in  $E_8 \times E_8$  through symmetric orbifolds cannot possibly work.

Alternatively, we can try to start with models constructed from  $Spin(32)/Z_2$  such as the  $SU(5)^3$  and  $SO(10)^3$  models obtained in section 3 by using the five-fold embedding. Again, the requirements on the gauge shifts and level matching conditions are usually very restrictive. For example, it can be proved with complete generality that, in the  $Z_2 \times Z_2$  orbifold, conditions of type (35) ( $j = 1$  in this case) and level-matching can only be consistent when either 4 or 8 pairs of coordinates are permuted. This is valid even if the permutation is accompanied by an arbitrary gauge shift. In order to obtain an  $SU(5)$  or an  $SO(10)$  GUT, the modding of 5 pairs of coordinates is necessary. Hence, this realization is ruled out. Moreover, this proof may be extended to all those even orbifolds where the  $Z_2$  sector of the orbifold feels the Wilson line permutation.

A way out to this limitation may be found in some cases. For example, this restriction may be avoided in a  $Z_4$  orbifold realized, either by using Coxeter rotations on the  $SU(4)^2$  root lattice or by assigning the permutation line to a third lattice direction in  $SO(4)^3$ . In both cases the  $\theta^2$  sector does not split in the presence of an

<i>Sector</i>	$SO(10) \times SO(12) \times SU(2)$	$Q_1$	$Q_2$	$Q_3$	$Q_4$
$U_1, U_2$	2 (1,1,2)	1	0	-1	0
	2 (1,1,2)	0	1	1	0
$U_3$	(1,1,1)	1	1	0	0
	(1,1,1)	-1	-1	0	0
	(10,12,1)	0	0	0	0
$\theta$	4 (16,1,1)	1/4	1/4	0	0
$(\theta^2, 2V)$	2 (1,12,1)	1/2	1/2	0	0
	2 (10,1,1)	-1/2	-1/2	0	0
	2 (1,1,1)	1/2	1/2	0	$\pm 1$
	(1,1,2)	1/2	1/2	$\pm 1$	0
	(1,1,2)	-1/2	-1/2	$\pm 1$	0
$Osc.$	(1,1,1)	-1/2	1/2	0	0
	(1,1,1)	1/2	-1/2	0	0
$(\theta^2, 2V + 2L)$	(1,12,1)	0	0	$\pm 1$	0
	(10,1,1)	0	0	$\pm 1$	0
	2 (1,1,1)	0	$\pm 1$	-1	0
	2 (1,1,1)	$\pm 1$	0	1	0
	(1,1,1)	0	0	$\pm 1$	$\pm 1$
$Osc.$	2(1,1,2)	0	0	0	0

Table 5: Particle content and charges before modding by permutations.

order two Wilson line (see Appendix ). However, the second situation is excluded in practice, due to restriction (34).

Let us discuss a particular example. We start with the  $Z_4$  orbifold in the  $SU(4)^2$  lattice and embedding

$$V = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0, 0, \frac{1}{4}, \frac{1}{4}, 0, 0, 0) \quad (45)$$

In order to lower the number of generations we also turn on Wilson lines  $L_1 = L_2 = L_3 = L$  with the specific order four  $L$

$$L = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}) \quad (46)$$

The emerging gauge group is  $SO(10) \times SO(12) \times SU(2) \times U(1)^4$ . The massless spectrum is found using the projectors described in the Appendix. The results are shown in Table 5.

The sector  $\theta$  is split into four sub-sectors but we do not find any massless states in those with shift  $(V + nL)$ ,  $n = 1, 2, 3$ . Massless generations are only found in the  $(\theta, 0)$  sub-sector. In this way the Wilson line effectively reduces the number of generations. State multiplicities are determined using the projectors discussed in the Appendix.

Permutation modding of the first  $SO(10)$  factor with the  $SO(10)$  subgroup of  $SO(12)$

$$\begin{aligned} \Pi(F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{11}, \dots, F_{16}) &= \\ (F_6, F_7, F_8, F_9, F_{10}, F_1, F_2, F_3, F_4, F_5, F_{11}, \dots, F_{16}) & \end{aligned} \quad (47)$$

may now be included as a Wilson line in the second  $SU(4)$  lattice. The surviving gauge group is therefore  $SO(10)_D \times SU(2) \times U(1)^5$  and matter states will organize into its corresponding representations. For example in the untwisted sector we obtain,

$$2(1, 2) + 2(1, 2) + (1, 1) + (1, 1) + (1, 1) + (54, 1)$$

where  $SO(10)$  singlets are split according to the different  $U(1)$  charges. The 54 of  $SO(10)$  is found (see discussion in section 4) when non invariant states are projected out from the starting  $(10, 12, 1)$  representation.

The twisted sectors in the initial model in Table 5 will split into sub-sectors that may or may not detect this second order permutation Wilson line. For example, the  $\theta$  sector includes a  $V$  sub-sector, corresponding to fixed points  $(0, 0)$  and  $(0, w_2)$ , not feeling the permutation line. There is also a  $V_{\Pi}$  sub-sector, corresponding to fixed points  $(0, w_1)$  and  $(0, w_3)$ , now feeling  $\Pi$ . Both sub-sectors contribute with a  $2(\overline{16}, 1)$  representation and therefore we end up with a four generation  $SO(10)$  model at level  $k = 2$ . Sixteen generations are found if the Wilson line  $L$  is not present.

As another example, let us mention that the third model of section 4, built up through the automorphism (33) plus the addition of a continuous Wilson line, may be reobtained by permutation modding. In fact, this is achieved by considering the shift  $V = \frac{1}{4}(1, 1, 2, 2, 2, 2, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0)$  which leads to an  $SO(14) \times SO(14) \times U(1)$  gauge group at level 1, and then adding a permutation Wilson line which exchanges the  $SO(10)$ 's contained in each  $SO(14)$ .

As the above examples show, the explicit models found are quite similar to the ones obtained through continuous Wilson lines. However not all the models obtained through permutation modding should be obtainable through the first method because the permutation modding involves a discrete projection on permutation-invariant states.

## 6 Constructing level two models from Higgs mechanism through flat directions

In Refs. [18, 20] it was remarked that the mechanism of gauge symmetry breaking through continuous Wilson lines may be understood perturbatively in terms of flat directions in the scalar potential of massless charged *untwisted* fields. In [6] it was shown that in fact there are flat directions which continuously connect level one to higher level string theories and an explicit  $k = 3$  example was constructed. More generally, higher level theories may be obtained by giving vevs along flat directions to *both twisted and untwisted* massless scalars [20]. The general procedure uses 4-D supersymmetry in the effective field theory to impose the flatness conditions

$$\begin{aligned} \langle W \rangle &= 0 \quad ; \quad \langle F_i \rangle = \langle \frac{\partial W}{\partial \phi_i} \rangle = 0 \\ \langle D_\alpha \rangle &= \langle g_\alpha \phi_i^* (T_\alpha)_j^i \phi^j \rangle = 0 \end{aligned} \tag{48}$$

where  $W$  is the superpotential,  $\phi_i$  the scalar fields and  $g_\alpha$  and  $T_\alpha$  the couplings and generators of the gauge group.

In the presence of an anomalous  $U(1)_A$  whose anomaly is canceled through a Green-Schwarz mechanism [27], there is a one-loop modification to the  $D$ -term.

This is a dilaton-dependent Fayet-Illiopoulos term which has to be added [28]

$$D_A = \sum_i q_A^i |\phi_i|^2 + \frac{g}{192\pi^2 \sqrt{k_A}} \text{Tr} Q_A \quad (49)$$

where  $\text{Tr} Q_A$  is the trace of the anomalous  $U(1)_A$  over the complete massless spectrum,  $g$  is the gauge coupling constant and  $k_A$  is the normalization (“level”) of the  $U(1)_A$ . In the following discussion we obviously assume that the value of  $g$ , determined by the dilaton vev, has been fixed by some non-perturbative dynamics that will not be discussed here.

Notice that for the usual classical vacuum  $\langle \phi_i \rangle = 0$  this extra term would induce supersymmetry breaking because  $\langle D_A \rangle \neq 0$ . However, what normally happens is that some of the  $\phi_i$ s are forced to have a vev and cancel the one-loop piece. For this to happen it is crucial that there exist fields  $\phi_i$  in the massless spectrum with charge  $q_i$  of sign *opposite* to that of  $\text{Tr} Q_A$ . Although there is no general principle that guarantees the existence of such fields, the fact is that up to now a 4-D string in which this is not the case has not been found. Thus, in the presence of an anomalous  $U(1)_A$ , classical string vacua are generically unstable but there is typically a nearby minimum which constitutes a one-loop stable vacuum. As we shall show, it turns out that the Fayet-Illiopoulos term often plays an important role in the construction of our class of GUT models.

For our particular interest of building GUTs, we start with level one models with gauge group and massless chiral fields of the type

$$\begin{aligned} & SU(5) \times SU(5) \times G \quad ; \quad (5, \bar{5}), (\bar{5}, 5) \\ & SO(10) \times SO(10) \times G' \quad ; \quad (10, 10) \end{aligned} \quad (50)$$

Giving appropriate vevs with vanishing D-terms to the chiral fields, the duplicated groups are spontaneously broken to the diagonal subgroups  $SU(5)_D$ ,  $SO(10)_D$  which are realized at level two. Of course, it must also be checked that the F-terms also vanish, which is sometimes non-trivial. The fact that the level is increased to two is explainable since one knows that, when a group  $G^M$  is broken to  $G_{diag}$ , the coupling constant must be rescaled as  $g \rightarrow g/\sqrt{M}$ . In the string context this means that the original level is rescaled as  $k \rightarrow kM$  [6].

Starting with duplicated groups is not the only possibility, one can also start with a level one model with group e.g.  $SU(5) \times G$  where  $G \supseteq SU(5)$  and similarly for  $SO(10)$ . The first model discussed in section 4 may be understood as an example of this type since it starts with gauge group  $SO(10) \times SO(18) \times U(1)^2$ , before adding the continuous Wilson line. By giving appropriate vevs to a field  $(10, 18)$  present in the untwisted sector, the final theory is the level  $k = 2$   $SO(10)$  model displayed in Table 4. To simplify the discussion though we will focus on models with repeated gauge group factors.

We already mentioned a class of orbifold models that naturally leads to replication of gauge groups  $SU(5)$  and  $SO(10)$ , namely the models obtained through the five-fold embedding we discussed in section 3. Let us then examine some of these.

#### *Flat direction model I (FD-I)*

Consider the simplest five-fold embedding example based on the  $Z_2 \times Z_2$  orbifold. It may be equivalently defined through the embedding

$$A = \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right)$$

<i>Sector</i>	$k = 1 : SO(10) \times SO(10) \times SO(10)$	$Q_A$	$SO(10)_{k=2} \times SO(10)_{k=1}$	$Q_A$
$U_1$	(1, 10, 1)	+1		
	(1, 10, 1)	-1		
	(10, 1, 10)	0		
$U_2$	(10, 1, 1)	+1		
	(10, 1, 1)	-1		
	(1, 10, 10)	0		
$U_3$	(1, 1, 10)	+1	(1, 10)	+1
	(1, 1, 10)	-1	(1, 10)	-1
	(10, 10, 1)	0	(54, 1) + (1, 1)	0
$\theta$	3(16, 1, 1)	-1/2	3(16, 1)	-1/2
	( $\overline{16}$ , 1, 1)	+1/2	( $\overline{16}$ , 1)	+1/2
$\omega$	3(1, 16, 1)	-1/2	3(16, 1)	-1/2
	(1, $\overline{16}$ , 1)	+1/2	( $\overline{16}$ , 1)	+1/2
$\theta\omega$	3(1, 1, $\overline{16}$ )	-1/2	3(1, $\overline{16}$ )	-1/2
	(1, 16)	+1/2	(1, 16)	+1/2

Table 6: Particle content and charges of the model FD-I, before and after taking the flat direction.

$$B = \left(\frac{1}{2}, 0, 0, 0, 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0\right) \quad (51)$$

One can easily compute the massless spectrum with the help of the generalized GSO projector given in the Appendix. The multiplicities in the twisted sectors depend on the particular realization chosen for the  $Z_2 \times Z_2$  twists acting on the cubic lattice. With the choice leading to the projectors eq. (78), we obtain the results displayed in Table 6 under the title  $k = 1$ .

Notice that in the  $U_3$  sector there is a (10, 10, 1) multiplet whose field we denote by  $\phi^{ij}$ . A vev  $\phi^{ij} = V\delta^{ij}$  has vanishing D-term and breaks the symmetry to the diagonal  $SO(10)$ . It is also easy to prove that this field direction is F-flat. Indeed, the discrete right-moving  $Z_2$  R-symmetries or “H-momentum” selection rules [20, 22], forbid any self-coupling of untwisted fields. The renormalizable Yukawas in the  $Z_2 \times Z_2$  are of the form  $(U_1 \cdot U_2 \cdot U_3)$ ,  $(U_1 \cdot \omega \cdot \omega)$ ,  $(U_2 \cdot \theta \cdot \theta)$ ,  $(U_3 \cdot \theta\omega \cdot \theta\omega)$ , and  $(\theta \cdot \omega \cdot \theta\omega)$ . The resulting level two GUT model particle content is shown in the right part of the table and in fact corresponds to an alternative construction of the second model discussed in section 4. The model has an anomalous  $U(1)$  and an associated dilaton-dependent Fayet-Illiopoulos term. It can be made one loop stable by giving a vev to the field (1, 10) with charge  $q_A = +1$ . This breaks the level one  $SO(10)$  group but does not affect the level two  $SO(10)$ . Altogether the GUT has four 16 generations and appropriate Higgs fields to break  $SO(10)$  down to the SM. Although apparently the possible Higgs 10-plets get mass along the flat directions, residual light Higgs doublets may result for particular values of the (10, 10, 1) and (1, 1, 10) (see the discussion about the doublet-triplet splitting problem in the next section).

#### *Flat direction model II (FD-II)*

A  $SU(5)$  model may be obtained from the previous one by adding a discrete

Wilson line  $L$ . Specifically, we add  $L_1 = L_2 = L$  where

$$L = \frac{1}{4}(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 3) \quad (52)$$

The group is broken to  $SU(5)^3 \times U(1)^3 \times U(1)_A$  and the spectrum is modified substantially. As explained in the Appendix, the Wilson line is only detected in the  $\theta$ -twisted sector. The complete chiral spectrum of the  $k = 1$  model is shown in Table 7.

In this model the  $U(1)$  generator  $Q_A$  is anomalous whereas the other three  $U(1)$ s are anomaly free. Thus, the classical vacuum is unstable and we have to look for a nearby vacuum which is one-loop stable. Since  $Tr Q_A = -48$ , some field with *positive*  $Q_A$  must acquire a vev to stabilize the D-term. In general, several fields do acquire vevs to cancel all the D-terms. We now describe an interesting scenario. Let us denote by  $\eta_1$  the singlet in the  $(\theta, L)$  sector with positive  $Q_A$  charge and by  $\eta_2$  the singlet in the  $\theta\omega$  sector also with positive  $Q_A$  charge. Then, the following field direction leads to cancellation of all D-terms and F-terms in the scalar potential

$$\begin{aligned} |\eta_1|^2 &= \frac{g}{4\sqrt{2}\pi^2} M_{string}^2 \\ |\eta_2|^2 &= \frac{g}{8\sqrt{2}\pi^2} M_{string}^2 \\ Tr(\phi^2 - \bar{\phi}^2) &= \frac{5g}{16\sqrt{2}\pi^2} M_{string}^2 \end{aligned} \quad (53)$$

where  $\phi$  and  $\bar{\phi}$  denote respectively the  $(\bar{5}, 5, 1)$  and  $(5, \bar{5}, 1)$  fields in the  $U_3$  untwisted sector. A diagonal vev  $\phi_j^i = v\delta_j^i$  would spontaneously break the first two  $SU(5)$  factors down to a diagonal  $SU(5)$  model realized at  $k = 2$ . The unbroken gauge group at this level would be  $SU(5)_2 \times U(1)_{Q_1+Q_2} \times SU(5)$ . Depending on the value of the field  $\bar{\phi}$  there may be a direct breaking from  $SU(5)^2$  down to the standard model.

In the process of symmetry breaking some of the untwisted matter fields get a mass due to the existence of couplings in the  $Z_2 \times Z_2$  orbifold of the type  $U_1 U_2 U_3$ . The remaining massless states turn out to be

$$\begin{aligned} U_1 &: (5, 1)_{1,-1} + (\bar{5}, 5)_{-1,-1} \\ U_2 &: (\bar{5}, 1)_{-1,-1} + (5, \bar{5})_{1,-1} \\ U_3 &: (24, 1)_{0,2} + (1, 1)_{0,2} + (1, 1)_{0,-2} + (1, 5)_{0,0} + (1, \bar{5})_{0,0} \end{aligned} \quad (54)$$

where the sub-indices are the charges with respect to  $Q_1 + Q_2$  and  $Q_1 - Q_2$  respectively. On the other hand, the twisted sectors are essentially the same as in the level one model, with the representations decomposed in terms of the diagonal  $SU(5)$ . Thus we have here a four-generation  $SU(5)$  model with one adjoint 24 and several Higgs candidates. The couplings among these fields are of special interest for the doublet-triplet problem, as will be discussed in the next section. If the vevs of  $\bar{\phi}$  and  $\phi$  are of the same order, the  $SU(5) \times SU(5)$  symmetry could be spontaneously broken directly to the standard model. The natural scale for this to happen, as indicated by equation (53), would be approximately equal to  $\sqrt{5g^2/8\pi^2} M_{string}$ .

There are similar models based on the  $Z_2 \times Z_4$  orbifold. For example, the embedding

$$\begin{aligned} A &= \frac{1}{2}(1, 1, 1, -1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1) \\ B &= \frac{1}{4}(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0) \end{aligned} \quad (55)$$



$Sector$	$SU(5) \times SU(5) \times SU(5)$	$Q_A$	$Q_1$	$Q_2$	$Q_3$
$U_1$	$(1, 5, 1)$	1	0	1	0
	$(1, \bar{5}, 1)$	-1	0	-1	0
	$(5, 1, \bar{5})$	0	1	0	-1
	$(\bar{5}, 1, 5)$	0	-1	0	1
$U_2$	$(5, 1, 1)$	1	1	0	0
	$(\bar{5}, 1, 1)$	-1	-1	0	0
	$(1, 5, \bar{5})$	0	0	1	-1
	$(1, \bar{5}, 5)$	0	0	-1	1
$U_3$	$(1, 1, 5)$	1	0	0	1
	$(1, 1, \bar{5})$	-1	0	0	-1
	$(5, \bar{5}, 1)$	0	1	-1	0
	$(\bar{5}, 5, 1)$	0	-1	1	0
$(\theta, 0)$	$2(\bar{5}, 1, 1)$	-1/2	3/2	0	0
	$2(10, 1, 1)$	-1/2	-1/2	0	0
	$2(1, 1, 1)$	-1/2	-5/2	0	0
$\omega$	$3(1, \bar{5}, 1)$	-1/2	0	3/2	0
	$3(1, 10, 1)$	-1/2	0	-1/2	0
	$3(1, 1, 1)$	-1/2	0	-5/2	0
	$(1, 5, 1)$	1/2	0	-3/2	0
	$(1, \bar{10}, 1)$	1/2	0	1/2	0
	$(1, 1, 1)$	1/2	0	5/2	0
	$(\theta, L)$	1/4	-5/4	5/4	1/4
	$(1, 5, 1)$	-1/4	5/4	-1/4	-5/4
	$(\bar{5}, 1, 1)$	-1/4	1/4	-5/4	-5/4
$\theta\omega$	$(1, 1, 1)$	-3/4	-5/4	5/4	5/4
	$(1, 1, 5)$	-1/4	5/4	-5/4	-1/4
	$(1, \bar{5}, 1)$	1/4	-5/4	1/4	5/4
	$(5, 1, 1)$	1/4	-1/4	5/4	5/4
	$(1, 1, 1)$	3/4	5/4	-5/4	-5/4
	$(1, 1, 5)$	1/2	0	0	-3/2
	$3(1, 1, \bar{5})$	-1/2	0	0	3/2
	$3(1, 1, 10)$	-1/2	0	0	-1/2
	$(1, 1, \bar{10})$	1/2	0	0	1/2
	$3(1, 1, 1)$	-1/2	0	0	-5/2
	$(1, 1, 1)$	1/2	0	0	5/2

Table 7: Particle content and charges of the level one  $SU(5)^3$  model.

leads to the gauge group  $SU(5)^2 \times SO(10) \times U(1)^3$  and to the untwisted particle content

$$\begin{aligned}
U_1 &: (5, \bar{5}, 1)_{1,1,0} + (\bar{5}, 5, 1)_{-1,-1,0} + (1, 1, 10)_{0,0,1} + (1, 1, 10)_{0,0,-1} \\
U_2 &: (1, 5, 10)_{0,-1,0} + (5, 1, 1)_{1,0,1} + (5, 1, 1)_{1,0,-1} \\
U_3 &: (\bar{5}, 1, 10)_{-1,0,0} + (1, \bar{5}, 1)_{0,1,1} + (1, \bar{5}, 1)_{0,1,-1}
\end{aligned} \tag{56}$$

where the sub-indices correspond to the charges of the three  $U(1)$ s. One linear combination of the three  $U(1)$ s is anomalous and a Fayet-Illiopoulos term is generated. One can check that vevs to  $(5, \bar{5}, 1)$ ,  $(\bar{5}, 5, 1)$ , and the positively charged  $(1, 1, 10)$ s in the  $U_1$  sector, are D-flat. They trigger the spontaneous breaking of  $SU(5)^2$  down to an  $SU(5)$  GUT with one adjoint 24.

In the examples above the GUT Higgs field arises from the untwisted orbifold sector. Indeed this is the simplest case but, as emphasized in section 2, fields of the  $(5, \bar{5})$  form may also appear in a restricted class of twisted  $Z_4$  and  $Z_6$  sectors of some orbifolds. An example is obtained using again the  $Z_2 \times Z_4$  orbifold with the same  $A$  above but flipping the sign of the  $B$  shift in the entries 6–10, i.e., choosing  $B = \frac{1}{4}(1, 1, 1, 1, 1, -1, -1, -1, -1, -1, 0, 0, 0, 0, 0, 0)$ . In agreement with our general arguments that order 4 twists such as  $b = 1/4(1, -1, 0)$  are among the few that allow massless particles of type  $(5, \bar{5})$ , we find that the  $\omega$  sector now includes several copies of  $(\bar{5}, 5, 1)_{\frac{1}{4}, -\frac{1}{4}, 0}$ . Fields in the opposite representations appear in the  $\omega^3$  sector if one consider a version of the model with a discrete torsion phase. Vevs to one of these fields again leads to an  $SU(5)$  at level two and several 24s. In this case one has 15s in the untwisted sector. In fact by flipping signs in the  $B$  as above we go from a model with an adjoint in the untwisted sector and 15s in the twisted sectors to another model with 15s in the untwisted sector and adjoints in the twisted sectors.

There are of course many other models that can be built but we refrain from presenting further examples. We have also considered  $Z_N \times Z_M$  orbifold models with discrete torsion. When adding discrete torsion one goes from one string GUT to another completely different. In the  $Z_2 \times Z_2$  case the addition of discrete torsion constitutes a sort of mirror operation with sends families to antifamilies and viceversa in the  $SO(10)$  models constructed. Our main goal in this paper, however, is to explain the construction methods and to look for general patterns that could be common to many string GUT models. Some of these properties are discussed in the following section.

## 7 Phenomenological aspects of String-GUTs

In the previous sections we have constructed several different string-GUTs with gauge groups  $SU(5)$  and  $SO(10)$  by three different methods. Our purpose in this paper is to describe techniques employed and to try to single out generic properties of the resulting class of models. We have shown that the construction of  $k = 2$  string-GUTs is relatively easy. In particular, within the context of orbifolds the complete massless spectrum as well as all the quantum numbers may be computed without the need of computer help. Moreover, having all this information allows us to make a cross-check of the modular invariance of the theories obtained by verifying the cancellation of gauge anomalies which may require the Green-Schwarz mechanism at work.

It is certainly risky to extract general conclusions from a limited class of models. Some of the general patterns we find are probably just a consequence of the technique used and it is non-trivial to separate what is generic in string theory and what is just a technical property of the method. Nevertheless we think it is worth to give a summary of the properties of the string-GUTs found.

*i. Generic features of the  $SO(10)$  models*

Both  $SO(10)$  and  $SU(5)$  string GUTs may be constructed, although  $SO(10)$  seems to appear more easily when either continuous Wilson lines or permutations are used to build the specific model. The  $SO(10)$  GUTs all share similar properties: they have a single 54 Higgs multiplet coming from the untwisted sector to do the GUT breaking. There are no 45s. We believe that this is a rather common feature of *left-right symmetric*  $k = 2$  string GUTs. This may be partially understood with the help of Table 1. Indeed, both representations 45 and 54 of  $SO(10)$  contribute a large amount to the left-handed conformal weight  $h_{KM}$  in such a way that they are more likely expected in the untwisted sector of the orbifolds. Although 45s of  $SO(10)$  may in principle also appear in some  $Z_4$  or  $Z_6$  twisted sector, we have not found any example. In fact, in our approach, GUT Higgses descend from a  $(10, 10)$  multiplet of an underlying  $SO(10)^2$  at level  $k = 1$  and, since  $h_{KM} = 1$  for such a multiplet, it cannot be present in any twisted sector.

In principle, both 45s and 54s could arise from the untwisted sector. However, in all the three different methods discussed in the text the resulting  $SO(10)$  algebra at  $k = 2$  is realized as the diagonal sum of some  $SO(10)^2$ ,  $k = 1$ , subalgebra of  $SO(32)$ . This inhibits the presence of a 45 in the untwisted sector. Indeed, if the model is constructed by a permutation method, in the decomposition  $(10, 10) = 45 + 54 + 1$  the projection on permutation-invariant states will kill the antisymmetric 45 field. If the method used is any of the other two, one can understand it, to some extent, as a continuous Higgs mechanism in which  $SO(10) \times SO(10)$  is spontaneously broken to the diagonal subgroup by a  $(10, 10)$ . Now, the 45 broken generators are given mass by the antisymmetric piece of the  $(10, 10)$  and no massless 45 is then left. Instead, the  $54 + 1$  symmetric components remain massless. This explains the presence of one massless 54 in this type of models.

It is important to remark that the above arguments are no longer true in the case of left-right asymmetric strings such as asymmetric orbifolds. In this case, as we mentioned before, there exists the freedom of twisting the compactified right-movers while leaving untouched their left-handed counterparts. Therefore, in  $M_L^2$ , c.f. eq. (3), there will be no energy shift  $E_0 = 0$  so that 45s and 54s may also surface in any twisted sector. Indeed, in Ref. [29] we have constructed explicit asymmetric orbifolds models in which both 45s and 54s show up in twisted sectors. On the other hand, some of the simple features of orbifold strings disappear in the asymmetric case, including the possible interpretation of the 4-D string as a compactification of a 10-D heterotic string.

Besides the 54-plet, the  $SO(10)$  models do in general contain  $(16 + \overline{16})$  pairs and both combined can break the symmetry down to the SM. Candidates for Higgs doublets usually also appear inside abundant massless 10-plets. Notice that with a 54-plet of Higgs fields the natural intermediate scale symmetry is the Pati-Salam  $SU(4) \times SU(2)_L \times SU(2)_R$  symmetry.

*ii. Structure of the GUT-Higgs potential*

In the construction of good-old SUSY-GUTs, the existence of certain couplings driving a vev for the GUT-Higgs was instrumental. For example, in SUSY- $SU(5)$  one assumes the existence of terms in the superpotential [1] :

$$W_5 = \Phi_{24}^3 + M\Phi_{24}^2 \quad (57)$$

where  $\Phi_{24}$  is the adjoint GUT-Higgs. This leads to a potential with several degenerate minima corresponding to  $SU(5)$ ,  $SU(4) \times U(1)$  or  $SU(3) \times SU(2) \times U(1)$  symmetries.

In string theory there are no explicit mass terms: a particle is either massless or has a mass at the string scale. In this latter case it makes no sense to consider a particular massive field as part of the effective Lagrangian while neglecting many others, thus the explicit mass term is absent. We also find that in the class of left-right symmetric string-GUTs we have constructed the cubic term is also typically absent. In particular, in all models in which the GUT-Higgs is in the untwisted sector, e.g. the 54s in the  $SO(10)$  models or the 24s in the  $SU(5)$  flat-direction models, the GUT-Higgs fields behave as string moduli and do not have self-interactions at all. From the 4-D point of view this may be understood as a consequence of the discrete  $Z_N$  R-symmetries which originate on the right-handed part of the string. These also imply the absence of couplings such as  $((5, \bar{5})(\bar{5}, 5))^n$  in  $SU(5)^2$  GUTs or  $(10, 10)^{2n}$  in  $SO(10)^2$ .

If the GUT-Higgs originates in a twisted sector, c.f. the  $Z_2 \times Z_4$  example at the end of previous section, the GUT-Higgs does not need to behave as a string modulus. Although we have not found any example, self-interactions of the GUT-Higgs can exist in this case. However, the presence of GUT-Higgses in twisted sectors of left-right *symmetric* orbifolds is relatively uncommon, so one may say that in this type of string-GUTs the absence of self-couplings of the GUT-Higgs is quite generic. In the case of asymmetric orbifolds GUT-Higgses may appear easily in twisted sectors and hence they do not necessarily behave like string moduli.

The absence of explicit mass terms and in some cases even of cubic terms makes it difficult to obtain GUT-Higgs superpotentials with the best desirable phenomenological properties. In particular it will be hard to find string-GUTs in which the particle content below the unification scale is just that of the minimal SUSY-SM.

If there are no self-interactions for the GUT-Higgs fields, there will be some extra matter fields remaining in the massless spectrum after symmetry breaking. For example, upon  $SU(5)$  symmetry breaking by an adjoint 24, twelve out of the 24 fields remain massless. They transform as

$$(8, 1, 0) + (1, 3, 0) + (1, 1, 0) \quad (58)$$

under  $SU(3) \times SU(2) \times U(1)$ . In the case of  $SO(10)$ , the extra fields depend on the GUT-Higgs triggering symmetry breaking. If it is broken by a 54, the resulting group in a first step is  $SU(4) \times SU(2)_L \times SU(2)_R$  and the following particles remain light

$$(20, 1, 1) + (1, 3, 3) + (1, 1, 1) \quad (59)$$

After further symmetry breaking down to the standard model, e.g. through a  $(16 + \bar{16})$  pair, those fields transform as

$$(8, 1, 0) + (6, 1, -2/3) + (\bar{6}, 1, 2/3) + (1, 3, 0) + (1, 3, 1) + (1, 3, -1) \quad (60)$$

where the third entry gives now the hypercharge. If  $SO(10)$  breaking proceeds through a 45, the remnant fields would transform as

$$(8, 1, 0) + (1, 3, 0) + (1, 1, 0) + (1, 1, +1) + (1, 1, -1) . \quad (61)$$

We see that the different breakings give rise to different extra matter fields. Since these particles will have masses of the order of the weak scale, they will sizably contribute to the running of the gauge coupling constants. We have performed a one-loop analysis of the running of the gauge coupling constants and have found that, with the particle content of the minimal SUSY-SM plus the additional fields above, there is no appropriate gauge coupling unification in the vicinity of  $10^{15} - 10^{17}$  GeV. Typically  $\sin^2\theta_W$  is too large and  $\alpha_s$  is too small. Thus, this class of models cannot break directly to the SM at a large GUT unification scale. In the case of  $SO(10)$  an intermediate scale of symmetry breaking could improve the results.

Thus, we see that if there are no self-couplings of the GUT-Higgs we lose one of the motivations for going to string-GUTS, a simple (one-step) understanding of gauge coupling unification.

### iii. Doublet-triplet splitting and the scalar field moduli space

The most severe problem of GUTs is the infamous doublet-triplet splitting problem of finding a mechanism to understand why, for example, in the 5-plet Higgs of  $SU(5)$  the Weinberg-Salam doublets remain light while their coloured triplet partners become heavy enough to avoid fast proton decay [1] . The most simple, but clearly unacceptable, way to achieve the splitting is to write a term in the  $SU(5)$  superpotential

$$W_H = \lambda H \Phi_{24} \bar{H} + M H \bar{H} \quad (62)$$

and fine-tune  $\lambda$  and  $M$  so that the doublets turn light and the triplets heavy. Since there are no explicit mass terms in string theory this inelegant possibility is not even present. Another alternative suggested long time ago is the “missing partner” mechanism [30] . Formulated in  $SU(5)$  it requires the presence of 50-plets in the massless sector which is only possible for level  $k \geq 5$ , a very unlikely possibility [6, 7] .

A third mechanism, put forward in the early days of SUSY-GUTs, is the “sliding singlet” mechanism [31, 32] . This requires the existence of a singlet field  $X$ , with no self-interactions, entering in the mass term in eq. (62).  $W_H$  is then replaced by

$$W_X = \lambda H \Phi_{24} \bar{H} + X H \bar{H} . \quad (63)$$

The idea is that the vev of the 24 is fixed by the potential in eq. (57) but the vev of  $X$  is undetermined to start with, i.e. the vev “slides”. Now, once the electroweak symmetry is broken by the vevs of  $H, \bar{H}$ , the minimization conditions give  $\lambda \langle -\frac{3}{2}v \rangle + \langle X \rangle = 0$  where  $\text{diag}(\langle \Phi_{24} \rangle) = v(1, 1, 1, -3/2, -3/2)$ . In this way  $X$  precisely acquires the vev needed for massless doublets. This is in principle a nice dynamical mechanism but it was soon realized that it is easily spoiled by quantum corrections [33] . For example, once SUSY is broken the field  $X$  will generically get a mass  $m_X$ . If  $m_X^2$  is positive, a large vev  $\langle X \rangle = -\langle \lambda \Phi_{24} \rangle$  will be strongly disfavored. If  $m_X^2$  is negative, a large vev will be preferred, but in general not the one that gives massless doublets. An extra problem [34] for the sliding singlet is

that it was also shown that light, order weak scale, singlets coupling to  $H, \bar{H}$  do in general destabilize the hierarchy by giving a large *soft* mass to the Higgs doublet.

Interestingly enough, we have found that in string GUTs, couplings of the “sliding singlet” type are frequent, the main difference now being that the GUT-Higgs field also “slides”. In particular, this happens in models in which the GUT-Higgs is a modulus, as in some of the examples discussed in the previous sections. Consider for instance the  $SU(5)^3$  model whose spectrum is shown in Table 7. The untwisted spectrum of the corresponding level 2 GUT is shown in eq. (54). As we mentioned above the couplings of untwisted fields in the  $Z_2 \times Z_2$  orbifold are of the type  $U_1 U_2 U_3$  and the following terms appear in the superpotential:

$$W_X = (5, 1)_{1,-1} [(24, 1)_{0,2} + (1, 1)_{0,2}] (\bar{5}, 1)_{-1,-1} + \dots \quad (64)$$

Vevs of these fields are restricted by absence of an  $SU(5)$  D-term and also we know that  $|(1, 1)_{0,2}|^2 - |(1, 1)_{0,-2}|^2$  is fixed according to eq. (53). Otherwise these terms are just like those in the sliding-singlet mechanism and would in principle give rise automatically to doublet-triplet splitting, were it not for the difficulties of the mechanism mentioned above.

Since the scales of the  $SU(5)^2 \rightarrow SU(5)$ , signaled by  $\langle (1, 1)_{0,-2} \rangle$ , and  $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$  symmetry-breakings are relatively near, one may alternatively describe this model as a level one  $SU(5)^2$  model which is broken spontaneously to a *level two* standard model by the two fields  $\phi_j^i = (\bar{5}, 5)$  and  $\bar{\phi}_i^j = (5, \bar{5})$ . The sliding-singlet solution corresponds in this language to the vevs

$$\phi_j^i = \begin{pmatrix} v & & & & \\ & v & & & \\ & & v & & \\ & & & v & \\ & & & & v \end{pmatrix}; \quad \bar{\phi}_i^j = \begin{pmatrix} \bar{v} & & & & \\ & \bar{v} & & & \\ & & \bar{v} & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix} \quad (65)$$

Models of this sort have recently been studied by Barbieri et al. [35]. The main difference in our case is the absence both of mass terms and cuartic  $\phi^2 \bar{\phi}^2$  couplings as well as the presence of a Fayet-Illiopoulos term. In the case of Ref. [35] a vacuum as in eq. (65) above may be obtained by fiddling with the parameters in the potential. In our case the potential is flat and only non-perturbative effects could lift the degeneracy. From the 10 moduli  $\phi_i^i, \bar{\phi}_i^i, i = 1, \dots, 5$ , the two  $\bar{\phi}_4^4$  and  $\bar{\phi}_5^5$  should remain vev-less for the splitting to occur. If the sliding-singlet mechanism survived the quantum corrections, it seems that field configurations with massless Higgs doublets would be energetically preferred.

Analogous couplings may be found in the  $SO(10)$  models with a massless 54 behaving as a modulus. Take for example the first model discussed in section 4 whose massless spectrum is displayed in table 4. The singlets in the  $U_3$  sector  $S^0 = (1, 1)_{0,0}$ ,  $S^+ = (1, 1)_{0,1}$ ,  $S^- = (1, 1)_{0,-1}$  do also behave as moduli. Both these singlets and the 54 couple to the decuplets  $H^+ = (10, 1)_{0,1}$  and  $H^- = (10, 1)_{0,-1}$ . The sub-indices in all these fields refer to their  $Q$  and  $Q_A$  charges. It is easy to check that there are flat directions in this scalar moduli space in which the gauge symmetry is broken down to  $SU(4) \times SU(2) \times SU(2)$  and some of the doublets remain light whereas the colour triplets remain heavy (the symmetry is broken down to the SM through the vevs of the  $16 + \overline{16}$  pairs). Again, if the sliding-singlet argument were stable under quantum corrections, the regions in moduli-space in which there are light doublets would be energetically favoured.

As the above examples show, the appropriate language to describe the doublet-triplet splitting problem within the context of the above string-GUTs is in terms of the scalar moduli space of the model. At generic points in the moduli space there are no massless Higgs doublets at all, they are all massive. At some “multicritical” points in moduli space some Higgs fields become massless. This is very reminiscent of the behaviour of the moduli spaces of other well studied string moduli, those associated to the size and shape of the compact manifold usually denoted by  $T_i$ . It is well known that generically there are points in the  $T_i$  moduli space in which extra massless fields appear. This is also apparently the case of the moduli space associated to the dilaton complex field  $S$ . The problem of understanding the doublet-triplet splitting within this context would be equivalent to finding out why we are sitting on a region of moduli space in which massless doublets are obtained. It could well be that an appropriately modified version of the sliding-singlet mechanism is at work and that region of moduli space is energetically favoured.

#### iv. $SU(5)^n$ and $SO(10)^n$ GUTs

We have seen that some of the simplest string-GUTs are obtained by starting with a  $SU(5)^2$  or  $SO(10)^2$  model at  $k = 1$  and giving vevs to  $(5, \bar{5})$ ,  $(\bar{5}, 5)$  or  $(10, 10)$  fields. It is worth stressing that this type of structure is very natural from the point of view of 4-D strings obtained from the  $Spin(32)/Z_2$  heterotic theory. Groups with repeated factors, e.g.  $(SU(5) \times U(1))^3 \times U(1)_A$ , may easily appear as we showed in section 3. For particular models some  $SU(5) \times U(1)$  factor(s) may be enhanced to  $SO(10)$ . The required Higgs fields to break the GUT symmetry are always present in the untwisted sectors of the above orbifolds. Replicated GUTs have been recently studied from a different perspective in ref. [35]. Notice however that in our case the couplings for the GUT-Higgs fields are rather different and, in particular, there are no self-interactions.

There is a related class of GUT models which also deserves attention. One may also obtain higher level GUTs by starting with  $k = 1$  models with gauge group factors  $G_{GUT} \times \hat{G}$  such that  $G_{GUT} \subseteq \hat{G}$ . An example of this is the first model in section 4. This may be understood as a  $k = 1$  model with gauge group  $SO(10) \times SO(18) \times U(1)^2$  which is continuously broken to  $SO(10)_2 \times SO(8) \times U(1)^2$  through appropriate vevs of the multiplet  $(10, 18)$  in the untwisted sector.

Models at  $k = 1$  with gauge group  $SU(5)^3$  may be easily constructed. However it does not seem to be trivial to find flat directions breaking the symmetry to the diagonal  $SU(5)$  subgroup of the three factors. The reason is that F-term couplings between the untwisted fields  $(5, \bar{5}, 1)$ ,  $(\bar{5}, 1, 5)$  and  $(1, 5, \bar{5})$  are allowed.

#### v. The rôle of the one-loop Fayet-Iliopoulos term

As we have seen, a very common feature in string GUTs is the presence of a one-loop dilaton-dependent Fayet-Iliopoulos term whenever there is an extra anomalous  $U(1)_A$  in the theory. As it turns out, most of the models do have such anomalous  $U(1)$ s. The existence of the F-I term often has an important impact in the phenomenology because it forces some charged massless fields to get vevs thus inducing symmetry breaking.

We showed some examples in which the F-I term actually triggers the  $G^2 \rightarrow G$  symmetry breaking leading to level two GUTs, and even the breaking down to the standard model. This is potentially very interesting since the natural scale of

GUT symmetry breaking is then related to the string scale by one-loop factors, i.e.  $M_{GUT} \sim (1/8\pi)M_{string}$ . This result is however less appealing if indeed extra massless particles remain in the spectrum after symmetry breaking because then the computed unification mass will not be the one of minimal  $SU(5)$ , for instance.

The effects of the F-I term may not always be desired. Sometimes its presence spoils some otherwise interesting tree level vacua. For example, we have seen that models with three GUT factors like  $SU(5)^3$  or  $SO(10)^3$  can be easily built. However, in some of these models F-I terms may force the breaking of at least one of the GUT factors.

## 8 Final comments and outlook

In this paper we have tackled the construction of standard  $SO(10)$  and  $SU(5)$  GUTs from 4-D string theories. One of our motivations has been to explore whether the effective low-energy limit of these 4-D strings resemble the well-known SUSY-GUTs introduced more than ten years ago or rather, string SUSY-GUTs have some specific properties on their own. The success of the SUSY-GUTs prediction for  $\sin^2\theta_W$  makes this exploration, in our opinion, worth pursuing. Our study requires the construction of 4-D string models in which the GUT gauge group is realized at high ( $k > 1$ ) level, otherwise the Higgs fields necessary to break the grand unified symmetry cannot be in the string spectrum.

In our approach, string GUTs are built by employing orbifold techniques. Within this scheme the massless spectra of the models may be easily computed without the need of computer help. The quantum numbers of the massless particles can also be determined and the cancellation of all anomalies can be verified. This provides a useful cross-check of the modular invariance of the models. One can also combine the structure of the string-GUTs so obtained with several phenomenologically interesting results, available in the orbifold context, such as one-loop corrections to coupling constants, SUSY-breaking soft term computations, etc..

In the present article we have concentrated on the case of symmetric  $(0, 2)$  orbifolds and have left the consideration of more general cases including asymmetric orbifolds for future work. Thus, the 4-D strings we are constructing may be understood as compactifications of the 10-dimensional  $Spin(32)/Z_2$  and  $E_8 \times E_8$  heterotic strings. To derive our models we have had to extend some of the known results about Abelian orbifolds in the presence of discrete Wilson lines. In particular, there are some subtleties concerning the generalized GSO projection in the presence of Wilson lines which are discussed in the appendix. We also discuss some aspects of the dependence of the spectra of  $Z_M \times Z_N$  orbifolds on the choice of underlying compactified lattice.

Three different methods, developed in a previous work, are used to build our models. The first one involves turning on continuous Wilson lines when the orbifold twist is realized in the gauge degrees of freedom by an automorphism of the gauge lattice. The second method uses the possibility of embedding discrete (order 2) Wilson lines as a permutation of two gauge groups of an original  $k = 1$  model. Finally, a third method considers flat scalar field directions in which a (semisimple)  $k = 1$  gauge group is continuously broken to a subgroup involving diagonal generators which are realized at  $k = 2$ . There are some connections among these three methods and sometimes the same model may be obtained in several possible ways.



In the past, heterotic string compactifications have mostly been based on the  $E_8 \times E_8$  theory whereas the  $Spin(32)/Z_2$  theory has been consistently ignored to the point that it is very difficult to find  $(0, 2)$  examples of  $Spin(32)/Z_2$ -based compactifications in the literature. Interestingly enough, we find that the  $Spin(32)/Z_2$  heterotic theory is the natural starting point in the derivation of string GUTs. We also find that the replicated GUT groups  $SO(10)^3 \times U(1)_A$  and  $(SU(5) \times U(1))^3 \times U(1)_A$  are naturally embedded into the  $SO(32)$  gauge group. Indeed, some, although not all, of the models we have constructed may be understood as level one  $SU(5) \times SU(5)$  or  $SO(10) \times SO(10)$  GUTs which are spontaneously broken down to the standard model.

It is possible to make some general model-independent statements about what string sectors may give rise to the Higgs fields required for the GUT symmetry breaking. We find that within the context of symmetric orbifolds those sectors are very much constrained. For instance, we can show in all generality that the string  $SO(10)$  Higgs fields transforming as 45 or 54 may only appear either in the untwisted sector or in a very restricted class of order 4 or 6 twisted sectors. In fact, all the particular  $SO(10)$  models constructed have one 54-plet and no adjoint 45-plets. These constraints are in general relaxed when asymmetric orbifolds are considered. We remark that related work using the 4-D strings fermionic formulation should correspond to asymmetric orbifolds and hence there is no direct connection between the string-GUTs considered here and those of Ref. [8].

We have attempted to identify some generic features of the string GUTs obtained through our methods. In many of the examples the GUT-Higgs fields behave as string moduli, i.e., they have no self-interactions. This is in general the case when the GUT- reside in the untwisted sector of the orbifold, the most frequent case in our type of constructions. In any event, it seems that the absence of GUT-Higgs self-couplings will make rather difficult to find string-GUTs whose massless sector is just the MSSM. Typically, extra chiral matter fields, such as color octets and  $SU(2)_L$  triplets, will remain massless. Thus, the presence of intermediate symmetry-breaking mass scales will be required in order to be consistent with gauge coupling unification. Other generic feature which plays a rôle in the GUT symmetry breaking is the presence of one-loop Fayet-Illiopoulos terms.

One of the toughest problems of GUTs in general is the famous doublet-triplet splitting problem of the Higgs system. We find that couplings of the “sliding-singlet” type are often present in the superpotential. In stringy language, we find that there are regions in scalar field moduli space in which there are light Higgs doublets and heavy scalar triplets. If the sliding-singlet mechanism were at work, those regions in moduli space would be energetically favored. This mechanism was shown to be generically destroyed by quantum corrections in the old SUSY-GUT days. It remains to be seen whether strings provide any improvement over that situation. The fact that in our class of models the GUT-Higgs often lives in the untwisted sector of the orbifold which has enhanced  $N = 4$  supersymmetry could perhaps point in that direction. Recent findings on the quantum structure of scalar moduli spaces in extended supersymmetric models could also have an important bearing on this question.

We have only scratched the surface of the class of orbifold string models leading to SUSY-GUTs at low energies. Many avenues remain unexplored. We believe that the doublet-triplet splitting problem is a crucial issue and should be addressed in any model before trying to extract any further phenomenological consequences such

as fermion masses. It is also important to understand whether it is possible to build string GUTs in which the massless sector is just the MSSM, or else whether the existence of extra massless chiral fields is really generic. This would dictate the necessity of intermediate scales to attain coupling constant unification. All the models displayed have four generations, a result just due to the particular structure of the  $Z_2 \times Z_2$  and  $Z_4$  orbifolds which naturally yield even number of generations. We did not attempt any systematic search for three generation models. We leave the question as well as the construction of models based on asymmetric orbifolds to a future publication [29] .

### Acknowledgements

A.F. thanks the Departamento de Física Teórica at UAM and the ICTP-Trieste for their hospitality and support while this work was carried out. G.A. thanks the Departamento de Física Teórica at UAM for hospitality and the Ministry of Education and Science of Spain (Programa de Cooperación con Iberoamérica) and CONICET (Argentina) for financial support. A.M.U. thanks the Government of the Basque Country for financial support. This work has been also financed by the CICYT (Spain) under grant AEN930673.

# A Appendix

The derivation of the orbifold massless matter content is a two-step process. First, the states satisfying  $M_R = M_L = 0$  are found in each sector. Next, the orbifold generalized GSO projection is imposed. A practical introduction to the full procedure, for embeddings by shifts, is given in Ref. [22]. We refer the reader to Appendix A there for the notation and formulae that we will use in the cases of  $Z_2 \times Z_2$  and  $Z_4$  relevant to our discussion.

We first consider the case without Wilson lines in which there are twisted sectors just for each element  $g$  belonging to the Abelian point group  $\mathcal{P}$ . The embedding of  $g$  is given by a shift  $V_g$ . The multiplicity of states in the  $g$ -twisted sector is given by an expression of the form

$$D(g) = \frac{1}{|\mathcal{P}|} \sum_{h \in \mathcal{P}} \tilde{\chi}(g, h) \Delta(g, h) \quad (66)$$

where we have neglected the possibility of discrete torsion for simplicity. Here  $\Delta(g, h)$  are phases that depend on the sectors and the states. The coefficients  $\tilde{\chi}(g, h)$  are numerical factors that only depend on the sectors. For instance,  $\tilde{\chi}(1, h) = 1$ .

Analysis of the partition function from which  $D(g)$  is derived shows that  $\tilde{\chi}(g, h)$  can generally be written as

$$\tilde{\chi}(g, h) = \mathcal{F}(g, h) \mathcal{O}(g, h) \quad (67)$$

Here  $\mathcal{F}(g, h)$  counts the number of inequivalent solutions for the center of mass  $x_{cm}$  that must satisfy the equations

$$\begin{aligned} x_{cm} &= gx_{cm} + u \\ x_{cm} &= hx_{cm} + w \end{aligned} \quad (68)$$

where  $u, w$  are vectors in the internal six-dimensional lattice  $\Gamma$  with basis  $e_i$ . For example, as shown in [26], when  $h = 1$

$$\mathcal{F}(g, 1) = \left| \frac{N_g}{(1 - g)\Gamma} \right| \quad (69)$$

where  $N_g$  is the sub-lattice orthogonal to the  $g$ -invariant sub-lattice  $I_g$ . Notice that when  $N_g = \Gamma$ ,  $\mathcal{F}(g, 1) = \det(1 - g)$  is the number of fixed points of  $g$ . When  $I_g$  is non-trivial the partition function also includes an instanton sum. However, we wish to stress that the factor  $1/Vol I_g$  arising from Poisson resummation cancels against another  $Vol I_g$  corresponding to the integral over the invariant directions of the center of mass. For  $h \neq 1$  we do not have a general formula and a case by case examination of eq. (68) is needed.

The other term  $\mathcal{O}(g, h)$  comes from the oscillator piece in the partition function. Since there are no fractionally modded oscillators in the non-trivial directions in  $I_g$ , a factor has to be extracted out from the corresponding Theta-functions. More precisely,

$$\mathcal{O}(g, h) = \frac{1}{\det'_{I_g}(1 - h)} \quad (70)$$

where  $\det'_{I_g}$  means that the determinant is evaluated in the  $I_g$  directions with non-zero eigenvalue of  $(1 - h)$ .

Let us now include Wilson lines  $L_i$ . We recall [18] that in the case of embedding by shifts the  $L_i$  are always quantized. For instance, in a  $Z_N$  orbifold it must be that  $NL_i \in \Lambda_{16}$ . Also, the  $L_i$  are not all independent but  $\forall g \in \mathcal{P}$  must verify  $gL_i = \mathcal{M}_{ij}L_j$ , where  $\mathcal{M}_{ij}$  is the integer matrix in  $ge_i = \mathcal{M}_{ij}e_j$ .

When the  $L_i$  are added, a  $g$ -twisted sector splits into several sub-sectors according to how the fixed sets of  $g$  detect the Wilson lines. Suppose that a given fixed set, labelled by  $x_g$ , is such that  $(1-g)x_g = n_i(x_g)e_i$ . The shift associated to  $x_g$  is then  $[V_g + n_i(x_g)L_i]$ . Notice that when the extra shift  $n_i(x_g)L_i$  happens to belong to  $\Lambda_{16}$  the  $x_g$  are Wilson line-blind. Different  $x_g$ 's with the same extra shift satisfy the same mass conditions and can thus be grouped into a sub-sector in the spectrum.

The Wilson lines also modify the generalized GSO projector in each sub-sector as we now explain. To each individual  $x_g$  it is convenient to assign a pre-projector

$$D(g|x_g) = \frac{1}{|\mathcal{P}|} \sum_{h \in \mathcal{P}} \tilde{\chi}(g, h|x_g) \Delta(g, h|x_g) \quad (71)$$

whose ingredients we now discuss in practical terms. If  $x_g$  is not fixed by  $h$ ,  $\tilde{\chi}(g, h|x_g)$  vanishes and this  $h$  does not contribute to the sum. If  $x_g$  is fixed by  $h$ , it must be that  $(1-h)x_g = m_i(x_g)e_i$ . Then,  $\tilde{\chi}(g, h|x_g)$  is given by a formula such as (67) with  $\mathcal{F}(g, h)$  essentially being the number of inequivalent  $m_i(x_g)$ . In this case, the gauge shifts  $V_g$  and  $V_h$  appearing in  $\Delta(g, h|x_g)$  must include extra contributions given respectively by  $S_g(x_g) = n_i(x_g)L_i$  and  $S_h(x_g) = m_i(x_g)L_i$ . We will then use the more explicit notation

$$\Delta(g, h|x_g) \equiv \Delta(g, h|V_g + S_g(x_g), V_h + S_h(x_g)) = \Delta(g, h) \left| \begin{array}{l} V_g \rightarrow V_g + S_g(x_g) \\ V_h \rightarrow V_h + S_h(x_g) \end{array} \right. \quad (72)$$

Modular invariance constraints on the  $L_i$  result from their presence in the phases  $\Delta$ . For instance, a full shift  $(V_g + S_g)$  must satisfy a condition similar to (7).

Once the quantization and relations among the  $L_i$  are taken into account, it is generally found that several  $x_g$  have the same  $S_g$  up to  $\Lambda_{16}$  lattice vectors. We then define a sub-sector  $(g, S_g)$  with overall projector

$$D(g, S_g) = \sum_{x_g | n_i(x_g)L_i \equiv S_g} D(g|x_g) \quad (73)$$

These issues will be clarified in specific examples below.

## A.1 $Z_2 \times Z_2$

We take  $\Gamma$  to be the hypercubic  $SO(4)^3$  lattice. The elements of the point group are  $\{1, \theta, \omega, \theta\omega\}$ . The corresponding shifts are

$$\begin{aligned} \theta : a &= (\tfrac{1}{2}, 0, -\tfrac{1}{2}) \longrightarrow A \\ \omega : b &= (0, \tfrac{1}{2}, -\tfrac{1}{2}) \longrightarrow B \\ \theta\omega : c &= (\tfrac{1}{2}, -\tfrac{1}{2}, 0) \longrightarrow C \end{aligned} \quad (74)$$

where  $2A, 2B \in \Lambda_{16}$  and  $C = A - B$ .

There are several allowed forms for the twists  $\theta$  and  $\omega$  written in a six dimensional basis  $(1, 0, 0, 0, 0, 0)$ , etc. [17]. To write these twists we use block notation, also  $\sigma_1$  is the  $2 \times 2$  Pauli matrix. One possibility is

$$\begin{aligned}\theta &= \text{diag} (-1, \sigma_1, \sigma_1) ; \omega = \text{diag} (-\sigma_1, -1, -\sigma_1) \\ \tilde{\chi}(\theta, 1) &= \tilde{\chi}(\theta, \theta) = 4 ; \tilde{\chi}(\theta, \omega) = \tilde{\chi}(\theta, \theta\omega) = 2\end{aligned}\tag{75}$$

Among others, another possibility is

$$\begin{aligned}\theta &= \text{diag} (-1, 1, -1) ; \omega = \text{diag} (1, -1, -1) \\ \tilde{\chi}(\theta, 1) &= \tilde{\chi}(\theta, \theta) = \tilde{\chi}(\theta, \omega) = \tilde{\chi}(\theta, \theta\omega) = 16\end{aligned}\tag{76}$$

In the following we will take choice (75) since it leads to lower multiplicities.

In this orbifold the untwisted sector splits in three sub-sectors denoted  $U_i$ . The corresponding projections are

$$\begin{aligned}U_1 : \quad & P \cdot A = \frac{1}{2} + \text{int} , \quad P \cdot B = \text{int} \\ U_2 : \quad & P \cdot A = \text{int} , \quad P \cdot B = \frac{1}{2} + \text{int} \\ U_3 : \quad & P \cdot A = \frac{1}{2} + \text{int} , \quad P \cdot B = \frac{1}{2} + \text{int}\end{aligned}\tag{77}$$

Each allowed state appears in one copy.

The multiplicity of states in each twisted sector is obtained by substituting (75) into (66). The results are

$$\begin{aligned}D(\theta) &= \frac{1}{2} [2 + 2\Delta(\theta, \theta) + \Delta(\theta, \omega) + \Delta(\theta, \theta\omega)] \\ D(\omega) &= \frac{1}{2} [2 + \Delta(\omega, \theta) + 2\Delta(\omega, \omega) + \Delta(\omega, \theta\omega)] \\ D(\theta\omega) &= \frac{1}{2} [2 + \Delta(\theta\omega, \theta) + \Delta(\theta\omega, \omega) + 2\Delta(\theta\omega, \theta\omega)]\end{aligned}\tag{78}$$

where  $\Delta(\theta^k \omega^l, \theta^t \omega^s) = \Delta(k, l; t, s)$  in the notation of [22].

Now consider turning on the Wilson lines  $L_1 = L_2 = L$  with  $2L \in \Lambda_{16}$ . The  $\theta$  sector splits into sub-sectors with embeddings  $A$  and  $A + L$ . In the  $(\theta, 0)$  sector the fixed sets are  $x_1 = (0, 0) \otimes (\alpha, \alpha) \otimes (\beta, \beta)$  and  $x_2 = (\frac{1}{2}, \frac{1}{2}) \otimes (\alpha, \alpha) \otimes (\beta, \beta)$ . They are fixed by  $\omega$  and  $\theta\omega$  for the particular values  $\alpha = 0, \frac{1}{2}$  and  $\beta = 0, \frac{1}{2}$ . The overall projector then turns out to be

$$D(\theta, 0) = \frac{1}{2} [1 + \Delta(\theta, \theta|A, A) + \Delta(\theta, \omega|A, B) + \Delta(\theta, \theta\omega|A, C)]\tag{79}$$

In the  $(\theta, L)$  sector the fixed sets are  $x_3 = (\frac{1}{2}, 0) \otimes (\alpha, \alpha) \otimes (\beta, \beta)$  and  $x_4 = (0, \frac{1}{2}) \otimes (\alpha, \alpha) \otimes (\beta, \beta)$ . They are not fixed by either  $\omega$  or  $\theta\omega$ . Hence,

$$D(\theta, L) = \frac{1}{2} [1 + \Delta(\theta, \theta|A + L, A + L)]\tag{80}$$

Finally, the sectors  $\omega$  and  $\theta\omega$  do not split at all and furthermore, the projectors remain those given in eq. (78).

## A.2 $Z_4$

We take  $\Gamma$  to be the product of two  $SU(4)$  root lattices. The point group is generated by the order four element  $\theta$  realized with shifts

$$\theta : v = \left(\frac{1}{4}, \frac{1}{4}, -\frac{1}{2}\right) \longrightarrow V \quad (81)$$

In each  $SU(4)$ ,  $\theta$  is represented by the Coxeter rotation given by

$$\theta_c = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix} \quad (82)$$

when written in the root basis  $\{e_1, e_2, e_3\}$ . It is easy to see that  $\theta_c$  fixes the origin  $w_0 = 0$  plus the three fundamental weights  $w_1, w_2, w_3$ . Hence,  $\theta$  has altogether 16 fixed points that are also fixed by  $\theta^2$ . We then find

$$\tilde{\chi}(\theta, 1) = \tilde{\chi}(\theta, \theta) = \tilde{\chi}(\theta, \theta^2) = \tilde{\chi}(\theta, \theta^3) = 16 \quad (83)$$

The sector  $\theta^2$  is subtler since there are fixed tori. Indeed,  $\theta_c$  has two fixed directions  $u_0 = \alpha(e_1 + e_3)$  and  $u_1 = w_1 + \alpha(e_1 + e_3)$ . Both are fixed by  $\theta_c$  provided  $\alpha = 0, \frac{1}{2}$ . This is another way of saying that  $\theta_c$  and  $\theta_c^2$  simultaneously leave fixed the four points  $w_i$ . Then,

$$\tilde{\chi}(\theta^2, 1) = \tilde{\chi}(\theta^2, \theta^2) = 4 ; \quad \tilde{\chi}(\theta^2, \theta) = \tilde{\chi}(\theta^2, \theta^3) = \frac{16}{4} = 4 \quad (84)$$

where we have used that  $(e_1 + e_3)$  is eigenvector of  $(1 - \theta_c)$  with eigenvalue 2.

The projections in the untwisted sector are

$$\begin{aligned} U_1, U_2 : \quad P \cdot V &= \frac{1}{4} + \text{int} \\ U_3 : \quad P \cdot V &= \frac{1}{2} + \text{int} \end{aligned} \quad (85)$$

Each allowed state has multiplicity one. The multiplicity of the twisted sectors is obtained by substituting (83) and (84) in (66). For example,

$$D(\theta^2) = 1 + \Delta(\theta^2, \theta) + \Delta(\theta^2, \theta^2) + \Delta(\theta^2, \theta^3) \quad (86)$$

This result was also obtained in Ref. [36] by slightly different arguments.

Let us now turn on Wilson lines. These can be of order two or order four. We will analyze the latter case. We then consider  $L_1 = L_2 = L_3 = L$  with  $4L \in \Lambda_{16}$ . The  $\theta$  sector splits into sub-sectors with shifts  $V, V \pm L$  and  $V + 2L$ . The  $\theta^2$  sector splits into sub-sectors with shifts  $2V$  and  $2V + 2L$ .

The Wilson line also affects the generalized GSO projection, even when it does not appear in the shift. In order to make this issue clearer, let us show this projector explicitly in the  $(\theta^2, 0)$  sector. The fixed sets with no extra shift are  $u_0 \otimes u_i$  which, as mentioned before, are fixed by  $\theta$  for particular values of  $\alpha$ . For  $\alpha = 0$ ,  $S_\theta(u_0 \otimes u_i) = S_\theta(w_0 \otimes u_i) = 0$ . For  $\alpha = \frac{1}{2}$ ,  $S_\theta(u_0 \otimes u_i) = S_\theta(w_2 \otimes u_i) = 2L$ . We then find

$$\begin{aligned} D(\theta^2, 0) &= \frac{1}{4} \{ 2 + [\Delta(\theta^2, \theta|2V, V) + \Delta(\theta^2, \theta|2V, V + 2L)] + 2 \Delta(\theta^2, \theta^2|2V, 2V) + \\ &\quad [\Delta(\theta^2, \theta^3|2V, 3V) + \Delta(\theta^2, \theta^3|2V, 3V + 2L)] \} \end{aligned} \quad (87)$$

The effect of  $L$  filtrates through phases  $\Delta$ .

## References

- [1] S. Dimopoulos and H. Georgi, Nucl.Phys. B193 (1981) 150; E. Witten, Nucl.Phys. B188 (1981) 513; N. Sakai, Z.Phys.C11 (1982) 153; For an introduction see G.G. Ross, "Grand Unified Theories" (Benjamin, New York, 1984).
- [2] H. Georgi, H. Quinn and S. Weinberg, Phys.Rev.Lett. 33 (1974) 451; S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D24 (1981) 1681 ; S. Dimopoulos and H. Georgi, Nucl.Phys. B193 (1982) 475; L.E. Ibáñez and G.G. Ross, Phys. Lett. 105B (1982) 439; M. Einhorn and D.R.T. Jones, Nucl. Phys. B196 (1982) 475.
- [3] J. Ellis, S. Kelley and D.V. Nanopoulos, Phys.Lett. B249 (1990) 441; B260 (1991) 131; P. Langacker and M. Luo, Phys.Rev. D44 (1991) 817; U. Amaldi, W. de Boer and H. Fürstenu, Phys.Lett. B260 (1991) 447.
- [4] For reviews see *String theory in four dimensions*, ed. M. Dine, North-Holland (1988); *Superstring construction*, ed. B. Schellekens, North-Holland (1989).
- [5] D. Lewellen, Nucl. Phys. B337 (1990) 61.
- [6] A. Font, L.E. Ibáñez and F. Quevedo, Nucl. Phys. B345 (1990) 389.
- [7] J. Ellis, J. López and D.V. Nanopoulos, Phys.Lett. B245 (1990) 375.
- [8] S. Chaudhuri, S.-w. Chung and J.D. Lykken, "Fermion masses from superstring models with adjoint scalars", preprint Fermilab-Pub-94/137-T, hep-ph/9405374; S. Chaudhuri, S.-w. Chung, G. Hockney and J.D. Lykken, "String models for locally supersymmetric grand unification", preprint hep-th/9409151. G.B. Cleaver, "GUTs from adjoint Higgs fields from superstrings", preprint OHSTPY-HEP-T-94-007, hep-th/9409096.
- [9] G. Aldazabal, A. Font, L.E. Ibáñez and A.M. Uranga, talks by L.E.I. in SUSY-94 (May 94, Michigan) and DESY SUSY Workshop (Sept. 94, Hamburg).
- [10] P. Ginsparg, Phys.Lett. B197 (1987) 139.
- [11] V. Kaplunovsky, Nucl.Phys. B307 (1988) 145 and erratum (1992).
- [12] L.E. Ibáñez, D. Lüst and G.G. Ross, Phys.Lett. B272 (1991) 251.
- [13] I. Antoniadis, J. Ellis, S. Kelley and D.V. Nanopoulos, Phys.Lett. B271 (1991) 31.
- [14] L.E. Ibáñez and D. Lüst, Nucl.Phys. B382 (1992) 305.
- [15] L.E. Ibáñez, Phys.Lett. B318 (1993) 73.
- [16] V.G. Kac, Funct. Anal. App. 1 (1967) 328; R.V. Moody, Bull. Amer. Math. Soc. 73 (1967) 217; K. Bardacki and M. Halpern, Phys. Rev. D3 (1971) 2493. For further references and recent reviews, see P. Goddard and D. Olive, Int. J. Mod. Phys. A1 (1986) 1 ; P. Ginsparg in *Fields, Strings and Critical Phenomena, Les Houches Summer School 1988*, eds. E. Brézin and J. Zinn-Justin, North-Holland.

- [17] L. Dixon, J. Harvey, C. Vafa and E. Witten, Nucl. Phys. B274 (1986) 285.
- [18] L.E. Ibáñez, H.P. Nilles and F. Quevedo, Phys.Lett. B187 (1987) 25; L.E. Ibáñez, J. Mas, H.P. Nilles and F. Quevedo, Nucl. Phys. B301 (1988) 137.
- [19] L.E. Ibáñez, H.P. Nilles and F. Quevedo, Phys. Lett. B192 (1987) 332.
- [20] A. Font, L.E. Ibáñez, H.P. Nilles and F. Quevedo, Nucl. Phys. B307 (1988) 109.
- [21] A. Font, L.E. Ibáñez and F. Quevedo, Phys.Lett. B217 (1989) 272.
- [22] A. Font, L.E. Ibáñez, F. Quevedo and A. Sierra, Nucl. Phys. B331 (1990) 421.
- [23] T.J. Hollowood and R.G. Myhill, Int. J. Mod. Phys. A3 (1988) 899.
- [24] L.E. Ibáñez, W. Lerche, D. Lüst and S. Theisen, Nucl. Phys. B352 (1991) 435.
- [25] T. Mohaupt, “Orbifold compactifications with continuous Wilson lines”, MS-TPI-93-09 (1993); G. Lopes-Cardoso, D. Lüst and T. Mohaupt, “Moduli spaces and target-space duality symmetries in (0,2)  $Z_N$  orbifold theories with continuous Wilson lines”, preprint HUB-IEP-94-6, hep-th/9405002.
- [26] K.S. Narain, M.H. Sarmadi and C. Vafa, Nucl. Phys. B288 (1987) 551; Nucl. Phys. B356 (1991) 163.
- [27] M. Green and J. Schwarz, Phys.Lett. B149 (1984) 117.
- [28] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B289 (1987) 585; J. Atick, L. Dixon and A. Sen, Nucl. Phys. B292 (1987) 109; M. Dine, I. Ichinoise and N. Seiberg, Nucl. Phys. B293 (1987) 253.
- [29] G. Aldazabal, A. Font, L.E. Ibáñez and A.M. Uranga, in preparation.
- [30] S. Dimopoulos and F. Wilczek, Sta.Barbara preprint, (1981); Proc. of the Erice summer school (1981); B. Grinstein, Nucl. Phys. B206 (1982) 387; A. Masiero, D.V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. 115B (1982) 380.
- [31] E. Witten, Phys. Lett. 105B (1981) 267.
- [32] L.E. Ibáñez and G.G. Ross, Phys. Lett. 110B (1982) 227.
- [33] See e.g. V. Kaplunovsky, Nucl.Phys.B233 (1984) 336.
- [34] H.P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. 124B (1983) 337.
- [35] R. Barbieri, G. Dvali and A. Strumia, Phys. Lett. 333B (1994) 79.
- [36] J. Erler and A. Klemm, Comm. Math. Phys. 153 (1993) 579.